Adapting assessment to instrumental genesis

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The development of information and communication technology together with its usage can be considered as instrumental genesis, triggered by instrumentation and instrumentalisation. The former means that a person can use the tool, whereas the latter means that the tool also shapes the actions and the character of the knowledge constructed with the tool. Mathematics assessment is still based on quite stereotypical tasks that very often are also a complement to the problems that exist in real life. This article argues why conventional task types cause a dead end in assessment when technology is implemented. It represents a framework to find new kinds of problems to be used for assessment independently of whether they are solved by using technology or not. This article version for TIME 2012 Proceedings is a summary of the article, submitted for The International Journal for Technology in Mathematics Education (IJTME).

Introduction

When considering the educational task of mathematics as Freudenthal (1974; 1991) does, we should see mathematics as mental art and emphasize active recognition, interpretation and solving of problems. Besides presenting the logical organization of mathematical knowledge, the focus should be on developing student's ability to construct and understand knowledge instead of merely collecting data. Teaching and assessment should also reflect the tools proved to be sustainable in the history of human thinking processes and for the generation of new mathematics. Zimmermann's (2003) long-term study of the history of mathematics reveals eight main activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years: order, find, play, construct, apply, calculate, evaluate, and argue. It sounds appropriate to take these activities as elements not only in a theoretical framework for structuring of learning environments or analyzing student’s cognitive and affective variables but also to assess the quality of mathematics teaching in general.

The results of Haapasalo & Eronen (2011) suggest that neither in Finland (Figure 1) nor in Germany does school mathematics seem to give support for those activities, university mathematics even less. The research objects were mathematics student teachers.

![Figure 1. Finnish (on the left) and German (on the right) mathematics student teachers’ average view of their competence in each of the activities (bold), of the support gained from school mathematics (thin), and from university mathematics (dashed).](image)

From several statements within our educational community one can recognise the message that it is in assessment where the battle of improving mathematics education will be won or lost. However, relating instructional design and assessment to instrumental genesis in modern society is far from an easy task. The author’s IJTME paper discusses this question as a challenge linked to six other challenges, originally represented in the author’s ATCM 2008 plenary (Haapasalo, 2008). This Proceedings article picks up just some remarks regarding those challenges.
From instrumentation to instrumentalization

I use the term ‘instrumental genesis’ to mean development of ICT together with its usage. ‘Instrumentation’ means that a person can use the instrument, whereas ‘instrumentalisation’ means that tool shapes the actions and the character of the knowledge constructed with the tool (Trouche, 2004). The predictive text input by typing cell phone messages, for example, can shape the user’s spelling perhaps more than school teaching. Whilst students in their free time are often on the level of instrumentalisation, educational institutions concentrate on instrumentation, performing trivial routines in their computer classes. The situation can be compared to the era when logarithmic tables were taught after they became unnecessary. The same holds for routines that have been possible to perform with CAS.

Instrumentation and instrumentalisation appear when using spreadsheets, CAS, dynamical geometry, dynamical statistics, CAD and other modelling/drawing, online databases, search engines and data mining, online-communication, supported learning environments (Learning Management), virtual environments, digital portfolios or other online assessment, and environments for reading, writing and publication. Many of these can be performed with hands-on technology. In addition to this, Internet forums, for example, very often promote and maintain a collaborative socio-constructivist working culture better than classroom work does. Individuals often set ambitious goals and commit to work to achieve them by donating their results to each other (cf. Schneiderman, 1998). Knowledge gained through those processes is not sterile without any transfer, but socially generated, viable knowledge that has both cognitive and pragmatic relevance. The evaluation of success is based on how the contribution of each individual will help the entire community to cope with a problem situation – i.e. the value of the donation - without any special external rewards, incentives, or stereotypical criteria. One such phenomenon is the trendy robotics, which has emerged around competitions, or children simply wanting to have fun among others by looking at the robot’s appearance and movements. Haapasalo & Samuels (2011) present a simple interaction between technology and mathematics via GeoGebra and show how such spontaneous research environments respond to the Challenges mentioned above.

By applying the so-called minimalist instruction at the very beginning of their so-called ClassPad project, Eronen & Haapasalo (2010) gave students in the 8th class the opportunity to play voluntarily during their summer holiday with some of the mathematical concepts of 9th grade by using a ClassPad calculator. This unfamiliar tool was briefly presented to them just a few days before their summer holiday. The only duty was to write a portfolio if they worked with the tool. The portfolio sample linked to Figure 2 (below the figure) shows that even a quite mediocre student showed good problem-solving competence by utilizing ClassPad properties in a sophisticated way without any tutoring from the teacher’s side.

Figure 2. An example of instrumentalisation with ClassPad.

- I draw a line (a). After drag and drop, the equation of the line is \( y = 1.613x - 0.5992 \) (b).
- By changing the equation to \( y = 2x - 0.5992 \) the angle between the line and y-axis is getting smaller (c).
- By changing the equation to \( y = 1x - 0.5992 \), the angle between the line and y-axis is getting bigger.
- I change the equation to \( y = 1.613x - 0.4 \). I don’t see any changes (in the graphic window).
- I change the equation to \( y = 1.613x + 4 \), the line moves to the same direction away from origin (d).
- When changing the equation to \( y = 1.613x + 4 \), the line moves in the same way, but to another direction on the x-axis with equal distance from the origin.
- I will continue in the morning. Time is now 1:42 a.m.
I worked 1 h 15 min.
The portfolio example refers to the potential of using hands-on technology and promoting sustainable mental activities. It reveals that by manipulating the equation (conceptual knowledge) spontaneously, the student explained how the parameters affect the position and location of the line (procedural knowledge). Through instrumentalisation the pupil made her own interpretation against the standard view: the line moves along the x-axis. This interpretation appeared also among other students when they studied the whole 9th grade mathematics curriculum using only the ClassPad calculator without any textbook or traditional homework. In addition to the fact that the cognitive results were clearly higher than in traditional teaching, pupils' mathematical profiles were extended when they were measured by using the eight sustainable activities from the history of mathematics (see Eronen & Haapasalo 2010).

Unfortunately innovative technological tools are usually made for those who apply mathematics, and not for learning purpose. Therefore most of the examples on the producers’ educational websites demonstrate rather instrumentation than instrumentalisation. Manuals often consist of several hundreds of pages containing huge amount of conceptual mathematical knowledge. This causes a contradiction between the versatility of the tool and minimizing the technical tutoring from the teacher’s side.

**Conceptual and procedural knowledge**

After an analysis of researchers’ views, Haapasalo and Kadijevich (2000) suggest the following characterization that fits viable theories of teaching and learning:

• *Procedural knowledge* (abbreviated here by *P*) denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.

• *Conceptual knowledge* (abbreviated by *C*, respectively) denotes knowledge of particular networks and a skillful “drive” along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

Based on the logical relation between these knowledge types, two pedagogical approaches are defined: *developmental* and *educational*. The first one is based upon the *genetic view* (i.e. *P* is necessary but not sufficient for *C*) or the *simultaneous activation view* (i.e. *P* is necessary and sufficient for *C*). The logical basis of the second one is the *dynamic interaction view* (i.e. *C* is necessary but not sufficient for *P*), or simultaneous activation view again. The latter means that the learner has opportunities to activate simultaneously conceptual and procedural features of the object.

Literature analysis reveals the dominance of *P* over *C* in the development of scientific and individual knowledge (see Haapasalo & Kadijevich, 2000; Zimmermann, 2003). We know from the basics of cognitive psychology that our world is a world of meanings, not a world of stimuli. When adapting a constructivist paradigm of teaching and learning, instead of 'learning environments' we should speak about 'investigation spaces' appearing psychologically meaningful for students. Today, an interactive manipulation at the computer screen can be more “real” to children than what is conventionally called “the real world”. This implies the need to apply a developmental approach in the instructional design: students should have opportunities to go for their more or less spontaneous procedural knowledge. On the other hand, perhaps the most important educational goal in a modern society is to scaffold citizens' abilities to identify and construct links within complicated multi-causal and multi-disciplined knowledge networks. This means investing in conceptual knowledge, even in such a way that students also learn appropriate procedural skills. Thus the educational approach seems to cause a conflict with the developmental approach. However, based on large empirical data and sophisticated statistical analysis, a brand-new dissertation by Lauritzen (2012) reveals that actually both approaches should be combined. The two knowledge types seem to develop iteratively, where a change of problem representation influences their relation. Such a development was assumed in the pedagogical model of the author’s *MODEM*-project (Haapasalo, 2007; 2008; and [http://wanda.uef.fi/lenni/modemeng.html](http://wanda.uef.fi/lenni/modemeng.html)). This quasi-systematic model for a sophisticated interplay between the two knowledge types makes use of spontaneous procedural knowledge as well as the simultaneous activation of conceptual and procedural knowledge within the philosophy of radical constructivism.

**Getting out from the deadlock caused by conventional task design**

To establish a systematic framework of task analysis, we can consider every problem to consist of the quadruple *Starting situation – Conceptual knowledge – Procedural knowledge – End situation*. Each of those components can be removed (Ø), given correctly (1), or given incorrectly (-1), defining a new type of problem in which the remaining components are present. This produces basically $3^3 \times 3 \times 3 = 81$ different categories. Table 1 shows some of those categories just to emphasize that even small changes in the way the problem is posed can shift the task into another category. The reader might be able to modify the examples so that some of the components are given incorrectly.
Even though in many countries the obstacles to using modern technology in both teaching and assessment have been removed at least formally, most tasks used in teaching and examinations are of a quite degenerated type: giving the starting point, concepts, and most often at least the name of the method (i.e. the first row in Table 1). The student has to find the end point, i.e. the correct solution. As regards technology, we have only the following three options when solving that type of task:

(i) it is not allowed to use technology,
(ii) technology may be used freely, or
(iii) technology must be used.

The first alternative represents the situation in many countries (as in Finland until the end of 2011), and in some of the sixteen federal states of Germany. Because very often a sophisticated tool such as graphic calculator can be used to get the result by pressing buttons couple of times, we must be honest in asking why all those mathematical paper-and-pencil tricks are still taught in school. On the other hand, the second alternative causes a serious conflict with the Equity Standard, emphasized by NCTM (1993), for example. Those who have the opportunity to buy sophisticated tools, namely, have the advantage in this kind of assessment tasks. The third alternative would not only make the situation strange, but also devalue the appreciation of sophisticated mathematical methods being applicable without technology. This would just shift the nature of “cookbook-teaching” or perhaps even accelerate it. We notice that the task type above is not only almost complementary for authentic problems occurring in real life but causes a dead end when used for assessment purposes. Still, it seems to be the most common one even in those countries that are pioneers in allowing the usage of technology in examinations. Creating an examination in two parts is a naive solution that simply multiplies the dead end by two. Because there are 81 different ways to get out of this deadlock, it would be appropriate to shift the focus on task types, which would be independent of the usage of technology. This would mean a thorough shift in mathematical assessment in general. The following simple prototypes, for example, fit for both teaching and assessment when using graphic calculator:

### Table 1. Classification of task types in assessment of mathematical knowledge including some examples.

<table>
<thead>
<tr>
<th>Starting point</th>
<th>Conceptual knowledge</th>
<th>Procedural knowledge</th>
<th>End point</th>
<th>Example</th>
</tr>
</thead>
</table>
| 1              | 1                    | 1                    | 0         | A quadratic polynomial \( P \) can be written in the form \( P(x) = ax^2 + bx + c \), where \( x \) is the variable and \( a, b \) and \( c \) are given real constants. The values of \( x \), where \( P(x) = 0 \), are called the roots of the polynomial. The roots can be found by the formula given below. Find the roots of the equation \( x^2 + x - 6 = 0 \).

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

| 1              | 0                    | 1                    | 1         | Assuming you see (e.g. on CAS screen) the equation \( ax^2 + bx + c = 0 \) and its two solutions:

\[
\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\]

Which mathematical concepts and which kinds of different representations are related to \( P \), if defined \( P(x) = ax^2 + bx + c \)? |
| 1              | 1                    | 0                    | 1         | A quadratic polynomial \( P \) can be written in the form \( P(x) = ax^2 + bx + c \), where \( x \) is the variable and \( a, b \) and \( c \) are given real constants. The values of \( x \), where \( P(x) = 0 \) are called roots of the polynomial and can be found by the formula

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Which kinds of methods could be used to get this formula? |
| 0              | 1                    | 1                    | 1         | The roots of a quadratic polynomial equation \( ax^2 + bx + c = 0 \) are \( 2 \) and \( -3 \). Assuming you know that any quadratic polynomial \( Q(x) \) can be written as \( Q(x) = (x - x_1)(x - x_2) \), where \( C \neq 0 \), and \( x_1 \) and \( x_2 \) are the roots of the equation \( Q(x) = 0 \), find at least three such of polynomials \( Q \) and draw the graphs. |
| 1              | 0                    | 0                    | 1         | Show as many methods as you can to find the roots \( 2 \) and \( -3 \) of the equation \( x^2 + x - 6 = 0 \). Which mathematical concepts and different forms of representations could you relate to the situation? |
Peter noticed that ClassPad was able to solve equations as $2x - 7 = 0$. He continued testing to see if the tool was able to solve more complex equations like $x^2 + x - 6 = 0$. Yes, it worked! Now Peter got excited about what the tool could actually do for $x^2 + x - 6$. He tapped menus for factorizing, simplifying, for example, and noticed there is an interesting connection between the roots 2 and -3 with $(x - 2)(x + 3)$. He continued playing with the tool and made a conjecture. What kind of conjecture do you think he made? Can you make your own hypotheses and perhaps see some mathematical concepts and procedures associated with the situation?

The function $f(x) := -x^2$, when $x < 0$, and $f(x) := 2x$, when $x \geq 0$. When using Texas CAS calculator, Lisa got the derivative $f'(x) = -2x$, when $x < 0$, and $f'(x) = 2$, when $x > 0$. Do you think Lisa or the calculator made a mistake by forgetting the value $x = 0$?

Remarks to other challenges

Taking the view that assessment should be considered within a continuum of planning, realizing and tracking of the teaching process, there are other important challenges to be taking into account when responding to the title of this article. First, the teachers should have knowhow of sustainable and viable frameworks for instructional praxis so that he or she would be able to scaffold students’ collaborative constructions toward viable knowledge. The MODEM framework, for example, emphasizes that a systematic planning of the instrumental orchestration is needed to promote heuristic processes and students’ ability to develop intuition and mathematical ideas. In learning situations, however, students should have the freedom to choose the problems that they want to solve within continuous self-evaluation instead of relying on guidance by the teacher. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. This challenge caused by so-called minimalist instruction, introduced by Carroll (1998), is discussed in the JJTME article.

Regarding sustainable heuristics in human history, teaching and assessment should reflect the tools that have been proved to be sustainable in the history of human thinking processes and for the generation of new mathematics. The ClassPad project revealed that doing mathematics voluntarily with a graphic calculator, even during a short period of time outside the classroom, enlarged 8th-grade students’ view of mathematics and also improved their self-confidence in performing the Zimmermann activities (see Eronen & Haapasalo, 2010). The portfolio sample was born in the spirit of Haapasalo (2007): “instead of speaking about ‘implementing modern technology into the classroom’ it might be more appropriate to speak about ‘adapting mathematics teaching to the needs of information technology in modern society’”. Very often the process of instrumentation and instrumentalisation allows the student to link conceptual and procedural knowledge. The ClassPad example also reinforces the viewpoint of learning by design: students should be seen as designers of their own lessons, whether ICT-based or not, rather than just as knowledge users.

As regards improving the bad reputation of mathematics, we might learn from the proverb “the customer is always right”. Why otherwise would a 14 year old girl be ready to spend 75 minutes to work with ClassPad in the middle of the most beautiful Finnish summer night to produce the portfolio linked to Figure 2? The same business principle was applied in the 9th grade when students learned mathematics in the class simply by using ClassPad without any textbooks and without mandatory homework. The problems for spontaneous investigations were planned within the framework of Table1 and the quasi-systematic model of MODEM, still allowing students to work within minimalist instruction.

Coda

It seems that both teachers and administrative bodies keep on doing what Howson et al. (1981) warned against: placing attention on what should be included in the curriculum for this or that school grade. It is not enough to add to this discussion the question of whether to allow the use of technology or not. Instrumental genesis has changed the essence of mathematics so radically that ‘instrumental orchestration’ and assessment should be considered from a new perspective (cf. Haapasalo 2008). Today almost every student in prosperous countries owns not only a mobile phone but also many other personal devices often representing more sophisticated technology than computer classrooms in schools can offer. It would be important to integrate these devices into mathematics learning. Many studies show that enjoyment and the possibility to be creative are seen as the key to affective motivation leading to an identity and attitude change towards mathematics (c.f. Loveless, 2002; de Freitas & Oliver, 2006; Harlen & Deakin Crick, 2003).

An integrated CAS and dynamic geometry, for example, might be one of the most fascinating combinations to trigger an environment for the construction of mathematical knowledge and to play with technology (cf. Haapasalo & Samuels, 2011). Instrumental genesis has already changed our views on making and teaching mathematics - and probably will change it even more radically. It might be quite evident that most of students’ instrumentation and instrumentalisation often happens in his or her free time especially when using hands-on technology. Thus, educators should shift their focus from well-prepared classroom lessons to scaffolding students by guiding them in recognizing, interpreting and solving problems. Instead of acting as a pace car in a race, institutions should be pit stops to promote students’ “race” outside the classroom. On the other hand, allowing the utilisation of technology in teaching and examinations does not necessarily improve students’ understanding and motivation if technology is merely used for conventional task types.
References


BIOGRAPHICAL NOTES

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