A Tool for Evaluating Solution Economy of Algebraic Transformations

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The extended abstract:

At our department, students solve exercises of the third-term course in Mathematical Logic on computers since 1991. In this paper we consider exercises on finding the disjunctive normal form of propositional formulae. For example, the program generates the formula ¬(A ∧ C) ∨ (B ∨ ¬AvB). The student should express implication and equivalence through negation, conjunction and disjunction, use De Morgan’s laws for pushing negations down to variables, expand the formula using distributive law, etc., until getting the full normal form A ∧ B ∧ ¬C v A ∧ ¬B ∧ ¬C v ¬A ∧ ¬B ∧ C.

Working in our formula manipulation environment, the student marks for each step a subformula and enters the result of conversion or chooses a conversion rule from the menu (29 rules available). The environment checks syntactical correctness (of marking and of input), equivalence between marked and entered subformulae, and applicability of selected rule. In case of an error the program issues a corresponding message and requires correction. As a result, recorded final solutions do not contain any direct mistakes. However, the environment does not check solution economy.

Some years ago the introductory part of propositional logic was moved into the first-term course in Elements of Discrete Mathematics (to improve students’ preparation for database and programming courses). We suddenly experienced that the solution files of many students appeared to be 3-4 or more times longer than normal. This was particularly noticeable in normal form exercises, which were solved in a rule-based working mode. An additional tool of solution analysis was implemented for fast identification of straying points in bulky solutions and for collection of statistics. The tool scans the solution and identifies the steps that do not correspond to the normal form algorithm. Inappropriate steps are classified by their nature into 15 types. Step displays and collected statistics are recorded in a text file for further observation. The tool also creates a table with statistics of the entire student group.

The presentation demonstrates our main environment and the analysis tool. We discuss the results of the final test of the course in Elements of Discrete Mathematics (162 participants, 132 completed and 30 partial solutions, 1249 suspicious steps diagnosed, the longest completed solution containing 269 steps). We also draw some conclusions for instruction and about functionalities that can be added to the main environment.

The keywords:
Algebraic transformations, normal forms, rationality of solution