Dear Colleagues, Old and New Friends,

It is a great pleasure to welcome you in the conference of *Technology and its Integration in Mathematics Education 2012* in Tartu. The conference has a long history and it has been held in different countries throughout the world.

The conference has changed its name several times from *Spring School Krems* 1992 to *International DERIVE & TI-92 Conference*, from *ACDCA Summer Academy* and *VISIT-ME 2002* to *TIME* since Montréal 2004. So we are proud to celebrate the 20th year of this conference series here in Tartu. No matter where it has been held, the conference has preserved a special spirit and atmosphere. So we hope that TIME 2012 will continue this tradition.

There are many people who have helped to organize this conference: chairs and members of the program committees, keynote speakers, presenters of lectures and workshops, members of the local organizing committee, etc. Many thanks to all of them!

We are also very grateful to our sponsors: Estonian Ministry of Education and Research, Texas Instruments, Tiger Leap Foundation and Faculty of Mathematics and Computer Science.

And finally special thanks to our families for their understanding and support.

Have a good and interesting TIME in Tartu!

Marina, Josef, Eno

The conference TIME 2012 (*Technology and its Integration in Mathematics Education*) combines 2 conferences:

12th ACDCA Summer Academy

and

10th Conference for CAS in Education & Research (Former *Int'l TI-Nspire & Derive Conference*)

The conference is hosted by the Institute of Computer Science, the University of Tartu, Estonia.

History and Scope

Brief History

In 1992, the ACDCA (Austrian Center for Didactics of Computer Algebra) began a conference series focusing on the use of symbolic calculation software in mathematics education. The first international conference dedicated to the use of Derive symbolic calculation software took place in 1994. After that time, ACDCA and Derive (to which the TI-92 symbolic calculator was added in 1996) conferences were held in alternating years. In 2002, the two conferences were merged into one for the first time, in Vienna, Austria. In 2012, Derive conferences are renamed as conferences for CAS in Education & Research.

2012 is a 20-Years' Celebration of this Conference Series.

Scope of conferences

The TIME conferences - founded by the ACDCA (Austrian Centre for Didactics of Computer Algebra) - deal primarily with didactical issues connected with the use of technology.

The conferences for CAS in Education & Research are geared towards exploring the use of CAS software and symbolic calculators in education (at the high school, lycée, college and university levels) and towards using these tools in programming and research.

These conferences are open to users of all symbolic/numeric systems (ie. not restricted to symbolic calculation as described). All interesting uses of technology related to mathematics education or innovative pedagogical approaches can form the basis of a conference lecture.

Topics include

12th ACDCA Summer Academy

- CAS-based curricula and teaching methods
- Dynamic geometry systems as teaching tools
- Assessing with technology
- Internet as a teaching aid
- Dispensable and indispensable mathematical skills and abilities
- CAS as pedagogical tools for compensation and amplification
- Demands on a Pedagogical CAS
- New classroom examples using CAS
- Future of CAS in Education
- The next generation: PeCAS = Pedagogical CAS
- CAS - Spreadsheet - Dynamic Geometry all in one?
- Compulsory use of ICT in classrooms

10th Conference for CAS in Education & Research

- Applications in mathematics, the natural sciences, research & development, economy, social sciences, industry, ...
- Programming in Computer Algebra Systems
- Interfaces to other programs/tools
- Producing and using utility files and libraries
- Problems and limitations
- Web resources - going online
- Connecting computer algebra and computational logic
- Recreational Mathematics supported by CAS and Dynamic Geometry
- Comparison of different computer algebra systems
- Modelling and Simulating Dynamic Processes

Chairs and Committees

Conference Co-Chairs

Eno Tõnisson, Josef Böhm, Marina Lepp

Organizing Committee

Eno Tõnisson, Marina Lepp, Tiina Lasn, Reelika Suviste, Anu Kuld, Sirje Pihlap, Triinu Arak, Kristi Kreutzberg, Tauno Palts, Liisi Reemets, Kristel Mikkor (excursions, program for accompanying persons)

Program Committee Co-Chairs

ACDCA Summer Academy

Josef Lechner, Austria
Marina Lepp, Estonia

Program Committee Members

Klaus Aspetsberger, Austria
Todd Edwards, USA
Wilfried Herget, Germany
Helmut Heugl, Austria
Rene Hugelshofer, Switzerland
Steve Joubert, South Africa
Djordje Kadijevic, Serbia
Zsolt Lavicza, United Kingdom
Rein Prank, Estonia
Eugenio Roanes Lozano, Spain
Evelyn Stepancik, Austria
Nurit Zehavi, Israel

The conference for CAS in Education & Research

Michel Beaudin, Canada
José Luis Galán, Spain

Program Committee Members

Gabriel Aguilera-Venegas, Spain
Steve Arnold, Australia
Jean-Jacques Dahan, France
Homero Flores, Mexico
Peter Hofbauer, Austria
Carl Leinbach, USA
Gilles Picard, Canada
Eugenio Roanes-Lozano, Spain
Pedro Rodríguez-Cielos, Spain
Karsten Schmidt, Germany
Elena Smirnova, USA
Agustín de la Villa, Spain

Proceedings

The selected papers will be published in the special issues of the following journals (in case the paper passes the review process):

Journal of Symbolic Computation (JSC),
International Journal of Mathematical Education in Science and Technology (iJMEST),
The International Journal for Technology in Mathematics Education (IJTME).

The paper submission deadline for the journals is September 15, 2012.

For further information, please check the conference website http://time2012.ut.ee/ or send an e-mail to José Luis Galán García at jl_galan@uma.es.

All other papers will be published in the conference webpage (and in downloadable CD).

Program

Tuesday, July 10

17.00-19.00	Registration (the Institute of Computer Science, Liivi 2 lobby)
19.00-21.00	Welcome Reception (the University of Tartu History Museum, Lossi 25)

Wednesday, July 11

	111	203	405	404
8.00-9.00	Registration (the Institute of Computer Science, Liivi 2 lobby)			
9.00-9.15	Opening Ceremony			
9.15-10.30	Plenary Lecture *P04* W. C. Bauldry			
10.30-11.00	Tea/Coffee Break (Liivi 2 cafe, lobby)			
11.00-11.55	*P10* J. Böhm	Workshop *P40* A. H. Flores Samaniego	*P06* M. Beaudin, G. Picard, G. Savard, C. Trottier	*P24* H. Heugl
12.00-12.25	*P44* G. Stolz		*P21* R. Hasek	*P19* A. Garcia, F. Garcia, G. Rodriguez, A. de la Villa
12.30-14.00	Lunch (Gunpowder Cellar, Lossi 28)			
14.00-14.55	*P29* L. C. Leinbach	Workshop *P11* J. Böhm	*P28* J. Kurvits, M. Kurvits	*P02* P. Antrobus
15.00-15.25	*P22* R. Heinrich		*P36* D. Pence	*P50* H. Urban-Woldron
15.30-16.00	Tea/Coffee Break (Liivi 2 cafe, lobby)			
16.00-17.15	Plenary Lecture *P32* P. Luik			
18.00-19.30	Tartu Walking Excursion (excursion will start from the fountain “The Kissing Students” located at the Tartu Town Hall Square)			

Thursday, July 12

	111	203	405	404	403
9.00-10.15	Plenary Lecture *P18* J. L. Galán				
10.15-10.45	Tea/Coffee Break (Liivi 2 cafe, lobby)				
10.45-11.40	*P12* J. Böhm	Workshop *P31* L. C. Leinbach	*P09* M. Beaudin, G. Picard, G. Savard, C. Trottier	*P41* K. Schmidt	*P35* R. Peetsalu
11.45-12.10	*P16* W. Ellis, W. C. Bauldry		*P33* W. Moldenhauer	*P17* T. H. Fay	
12.15-13.15	Lunch (Liivi 2 cafe)				
13.15-20.00	Excursion to Lake Peipsi region or Alam-Pedja Nature Reserve (buses will depart near the institute building, Liivi 2)				

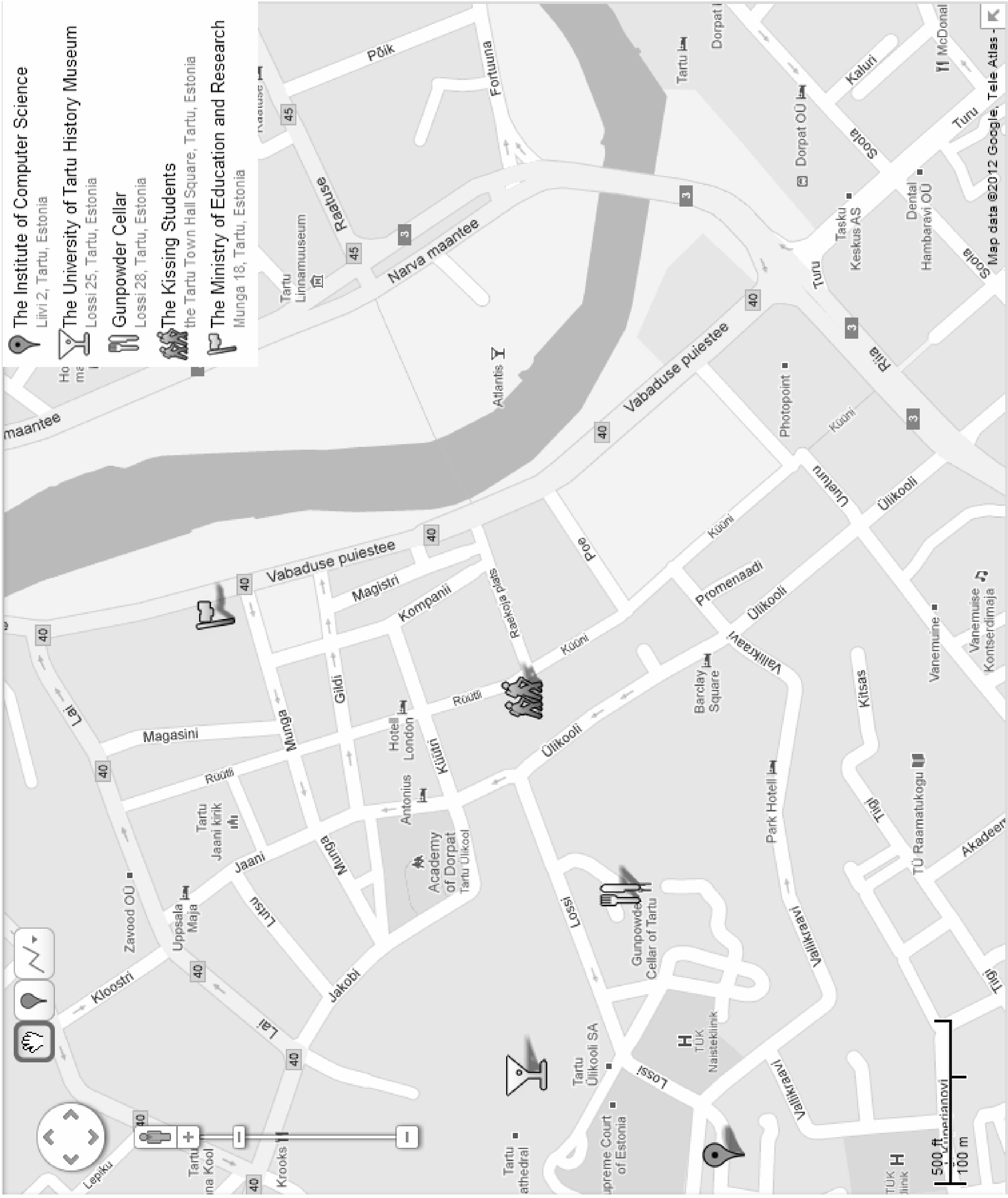
Friday, July 13

	111	203	405	404	403
9.00-10.15	Plenary Lecture *P26* D. Kadijevich				
10.15-10.45	Tea/Coffee Break (Liivi 2 cafe, lobby)				
10.45-11.40	Discussion panel *P03* W. C. Bauldry, W. Ellis, H. Flores, R. Heinrich, H. Heugl	Workshop *P13* J. Böhm	*P08* M. Beaudin, G. Picard, G. Savard, C. Trottier	*P14* E. Chepurko	
11.45-12.10			*P27* J. Kaljula	*P49* H. Urban-Woldron	
12.15-13.45	Lunch (Gunpowder Cellar, Lossi 28)				
13.45-14.40	*P20* L. Haapasalo		*P30* L. C. Leinbach	*P38* R. Prank	*P23* G. Herweyers
14.45-15.10	*P45* P. Todd		*P07* M. Beaudin	*P51* Q. van der Hoff, J.C. Greeff, T.H. Fay	*P37* S. Pihlap, S. Sild, K. Kreutzberg
15.15-15.40				*P01* G. Aguilera, J.L. Galán, A. Gálvez, Y. Padilla, P. Rodríguez, R. Rodríguez	
15.45-16.15	Tea/Coffee Break (Liivi 2 cafe, lobby)				
16.15-17.10	*P42* D. Sjöstrand		*P34* J. Oikarinen, R. Kaasila	*P25* S. V. Joubert, M. Y. Shatalov, C. E. Coetzee	Discussion panel *P48* E. Tõnisson
19.00-22.00	Banquet (the building of Estonian Ministry of Education and Research, Munga 18)				

Saturday, July 14

	111	203	405	404
9.00-10.15	Plenary Lecture *P52* W. Wegscheider			
10.15-10.45	Tea/Coffee Break (Liivi 2 cafe, lobby)			
10.45-11.40	Discussion panel *P39, P05* M. Beaudin, J. Böhm, J. L. Galan, C. Leinbach, E. Roanez-Lozano	Workshop *P46* P. Todd	*P15* J.-J. Dahan	*P47* E. Tõnisson
11.45-12.10			*P43* A. Stolyarevska	*P53* T. Westermann
12.15-12.45	Closing Ceremony			
12.45-13.45	Lunch (Liivi 2 cafe)			

The map with locations



Abstracts

P01: A Stepwise Graphical Approach for Teaching Automated Theorem Proving in Engineering

G. Aguilera, J.L. Galán, A. Gálvez, Y. Padilla, P. Rodríguez, R. Rodríguez
Universidad de Málaga
Spain

The extended abstract:

Automated theorem proving (ATP) for Propositional Classical Logic is an algorithm to check the validity of a formula. It is a very well known problem, which is decidable but co-NP-complete. There are many algorithms for this problem. In this talk, an educational oriented implementation of Semantic Tableaux method will be described.

The program developed has been designed to be used as a pedagogical tool to help in the teaching and learning process of Propositional Classical Logic subject. Therefore, the main goal of the implementation is not a great efficiency but to get a useful tool for education. For this reason, the implementation of Semantic Tableaux method provides an optional graphical approach, which shows, step by step, the trace of the algorithm when applied to a specific problem.

The algorithm has been implemented using a CAS (Computer Algebra System), specifically Derive, as part of the students training in Mathematics for Engineering. The result is a utility file containing a didactical implementation of Semantic Tableaux algorithm, which can be used not only at lectures but also as a self-learning tool by students. With this utility file, students can check the results obtained by hand and, in case they get a wrong result, students can find the step where the mistake is. Furthermore, if the student does not know how to solve an exercise, the utility file can show how to overcome this situation step by step.

On the other hand, since Derive does not have Semantic Tableaux algorithm implemented, this utility file increases the capabilities of this CAS. Therefore, Derive can act as an Automated Theorem Prover for Propositional Classical Logic from a PeCAS (Pedagogical CAS) point of view.

The description of how to use this utility file will be shown throughout different examples related to validity, satisfiability and deduction, which are the main kind of problems that an ATP can solve.

This work is part of our research line on the use of CAS in Mathematic Education in Engineering and it is supported by many previous works in this research line, including 3 PhD Thesis developed in the University of Málaga (Spain).

The keywords:
CAS, propositional classical logic, ATP, semantic tableaux, mathematics education

P02: Technology in Maths (a Queensland Perspective)

Peter Antrobus
Kawana Waters State College, Queensland Studies Authority
Australia

The extended abstract:

The session looks at the history of the use of technology in the Mathematics Classroom with specific examples of how they have been applied in the classroom and assessment. It examines the place of CAS calculators along with their Graphical predecessors. Student examples are shown and how they are developed. Some of the examples have been used as exemplars for teachers of Maths B and are currently posted on the Queensland Studies Authority website.

The keywords:
CAS calculators

P03: Assessment and Mathematical Placement Exams with Technology

William C Bauldry, Wade Ellis Jr., Homero Flores, Rainer Heinrich, Helmut Heugl
USA, USA, Mexico, Germany, Austria

The extended abstract:

An effective yield of the use of technology in mathematics education can only be expected if the tools are also applied accordingly in the exam situation. Discussing this topic we have to keep in mind on the one hand, the process oriented assessment as an integral part of the learning process, and on the other hand, product oriented mathematical placement exams and final exams as a proof of authority for the students.

In consideration of this fact several questions could be discussed like:

- Should we try to adapt technology supported exams to traditional exams which are mostly calculation oriented or should we think about new ways of assessment?
- How could we test mathematical fields which are at first available by technology?
- How should we deal with technology which knows too much (E.g. analyzing the graph in the graphic window provides zeros, maxima, integrals tangents, a.s.o without any mathematical activity of the students)?
- Should we regulate rules for the presentation and documentation of the problem solving process and the results and answers. Is it enough when students are documenting what keys they have pressed or should they still use a mathematical language? How many intermediate data should they document?
- Is it possible to take into account typical effects of technology supported learning like experimental learning or cooperative learning in classroom assessment and product oriented exams?

The five organizers, which have wide experience in the field of mathematics diagnostic testing, in developing and organizing final exams and in incorporating CAS in assessment, will at first present their position. Afterwards participants attending the session are invited to pose their questions and suggestions.

The objectives of the panel discussion will be to find first answers and solutions and to lay the basis for further discussions and research projects.

P04: CAS in the Classroom: Yesterday, Today, and Tomorrow

William C Bauldry
Appalachian State University
USA

The extended abstract:

We will consider the development of classroom use of computer algebra systems (CAS) from different perspectives. Starting with a brief history and timeline of CAS, we'll highlight the explosive growth during the '60s and 80's. The innovations of these two decades brought CAS to the classroom. Early uses were limited, but implementing CAS on microcomputers and the addition of good graphics fomented widespread adoption.

Current usage is our next topic. Today there are two main pedagogical practices: CAS as a high-level computation engine, and CAS-based microworld's for exploration. We are presently at the cusp between desktop and mobile platforms. Another quantum growth step in CAS adoption, like the mainframe-to-desktop burst of the 80s, is beginning. Programs and resources such as PocketCAS & iCAS and Maple Player & Wolfram Alpha are presenting new opportunities to incorporate CAS in our classrooms.

We will close with predictions and hopes for future CAS technology. The question is, "Where can CAS help us to go in didactics and pedagogy tomorrow?"

P05: DERIVE is Still Special

Michel Beaudin, Josef Böhm, José Luis Galan, Carl Leinbach
Canada, Austria, Spain, USA

The extended abstract:

At the TIME 2006 conference in Dresden, several DERIVE program users were shocked to hear that DERIVE would be discontinued and replaced by the new product TI-Nspire CAS. In 2007, a transition period started where users asked TI developers to implement some interesting features of DERIVE. TI-Nspire CAS, with two platforms and a wider spectrum - due mainly to numerous integrated applications -, could not satisfy all wishes but tried to address some of them: one particular example is found in the latest operating system of Nspire CAS (OS 3.2) where 3D parametric plotting has been added with David Parker on board. But there are still special and unique features of DERIVE that cannot be found in Nspire CAS.

In this panel discussion, we would like participants attending this session to talk about what they still find special in DERIVE: to do so, live examples in various fields of mathematics will be shown to the audience using DERIVE. One goal of this discussion is to provide constructive ideas to Texas Instruments developers: proclaimed at its beginning as the "true successor of DERIVE", Nspire CAS has all the potential needed to satisfy many requests of users.

Topics for discussion include, but are not limited to: Subexpressions, Showing Step Simplification, 2D implicit plotting, precision digits, cubic equations, piecewise continuous functions, Expand/Collect for trig expressions, creating random numbers without the randseed, plotting polyhedrons in 3D, iteration, Fourier series, polynomial systems, Gröbner bases. Those who would like to present one's favourite DERIVE special which has not been implemented in TI-Nspire so far are welcome to join.

P06: Adopting TI-Nspire CAS Technology (Handheld and Software), a Campus-Wide Experience

Michel Beaudin, Gilles Picard, Geneviève Savard, Chantal Trottier
Service des enseignements généraux, École de technologie supérieure
Canada

The extended abstract:

In September 2011, our engineering school has adopted the TI-Nspire CAS CX calculator (and software) for all new students entering bachelor level. Although we started to use this new device in the classroom, we will have a transition period where both the TI Voyage 200 and the new CX handheld calculators can be on the students desks. In order to be prepared for the new academic year, we had to launch a new support website and needed to organize workshops and seminars during summer and fall of 2011. This talk will report on many of these activities and on the steps taken to achieve a smooth transition with a widespread acceptation by teachers.

We will discuss in details the advantages in regards with the previous technology (which was very well implemented) and describe why students are now getting more "bang for their money". Having a software-handheld bundle is also a big plus; we want the handheld in the classroom for teaching and exams but, we also need the software (with the same commands and possibilities) for them to better explore some aspects, graphics for example. But this comes at a price; we will show why teaching the use of a handheld calculator with a software is not always an easy task.

We will present our support website and the multiple workshops offered to teachers and instructors, thus gaining interest even with science teachers, economics and project management colleagues. After 11 years of using TI-92 Plus/Voyage200, we had to show the new avenues introduced by additional functionalities and graphical capabilities (spreadsheet, animation, dynamic geometry) in order to convince them to move to this new technology. Classroom examples of its use will be presented with some advice on the do's and don'ts.

The keywords:
TI-Nspire CAS technology, calculator/software

P07: Why Try to Avoid Using Complex Numbers?

Michel Beaudin
École de technologie supérieure
Canada

The extended abstract:

The Complex Format of Voyage 200 and Nspire CAS defaults to "Real" instead of "Rectangular" ("Complex"). This choice can be easily justified for high school/college level mathematics up to single variable calculus. But for higher level mathematics, choosing the real branch has more inconveniences than advantages. The talk will relate some situations and examples of what happens when you have decided to switch from one branch to the other: these situations can occur when one tries to solve an equation or system of equations, a differential equation, when someone computes an indefinite integral and so on. Not surprisingly, the complex format environment gives the teacher the freedom to do more mathematics instead of less and the price to pay is small when you consider the benefits. It is true that the complex branch can cause bad surprises when you need to compute certain improper integrals: who said that everything should be perfect?!

The keywords:
CAS, equations, indefinite integrals, ODEs

P08: Using TI-Nspire CAS Technology in Teaching Engineering Mathematics: ODE's

Michel Beaudin, Gilles Picard, Geneviève Savard, Chantal Trottier
Service des enseignements généraux, École de technologie supérieure
Canada

The extended abstract:

In September 2011, our engineering school has adopted the TI-Nspire CAS CX calculator (and software) for all new students entering bachelor level. With both platforms and integrated facilities, Nspire CAS is becoming a more complete solution than Voyage 200. Although we already started to use this new device in the classroom, we will have a transition period where both the TI Voyage 200 and the new CX handheld calculators can be found on the students' desks. This talk will demonstrate examples of how this technology is used in classrooms when teaching differential equations.

Nspire CX handheld with its faster CPU and extended graphing capabilities gives us even more options for doing math in our classes. Starting with first order equations, the use of an Nspire-CAS calculator makes it easy for students to explore slope fields, numerical solutions with Euler's method or even Picard's approximations for well behaved equations. With Nspire, we can now see graphically how a numerical solution fits when compared to an exact solution. Consider applications of first order ODE's, Newton's law of cooling for example. With Nspire, students can hook up a temperature probe to their calculator and gather data which will then be compared to the theoretical solution.

Looking at second order equations, a CAS calculator can help in computing part of the algebraic solution. In the method of undetermined coefficients, we still teach the classical approach but let students use their handheld to substitute the candidate in the equation and solve the resulting system of equations. When teaching the method of variation of parameters, we show them step by step solution (with the help of their calculator) instead of giving them a unique formula for the solution.

For teachers, having this technology can be very useful for grading exams. One can construct a small program that will yield step by step solution for the variation of parameters solution. If a student makes an error at the beginning (wrong solutions for the homogeneous equation), the teacher could easily obtain what should be the remaining solution for this student. We will demonstrate how simple it is to create such a program.

Our students must learn skills in regards to technology: this is now part of the curriculum and will be presented. In exams or in take-home work, some problems will require the use of Nspire CAS (or Voyage 200) technology. Students should also learn to validate manual results using these tools and understand unexpected results obtained by CAS systems. We will present examples where the benefit of using this approach is obvious. Moreover, the majority of students will spend a lot of energy exploring these math concepts if these tools are used. Everything is more visual and they love to play with technology, as long as they are told which direction to follow.

The keywords:
TI-Nspire CAS technology, calculator, differential equations, numerical solutions, programming

P09: Using TI-Nspire CAS Technology in Teaching Engineering Mathematics: Calculus

Michel Beaudin, Gilles Picard, Geneviève Savard, Chantal Trottier
École de technologie supérieure
Canada

The extended abstract:

In September 2011, our engineering school has adopted the TI-Nspire CAS CX calculator (and software) for all new students entering bachelor level. With both platforms and integrated facilities, Nspire CAS is becoming a more complete solution than Voyage 200. Although we started to use this new device in the classroom, we will face a transition period where both the TI Voyage 200 and the new CX handheld calculators can be found on the students' desks. This talk will demonstrate examples of how this technology is used in classrooms for our courses in calculus.

Nspire CAS CX handheld with its faster CPU and extended graphing capabilities offers us even more options for doing math in our classes. Here are some topics where we can benefit from the use of this new CAS calculator in the single variable course: defining the derivative (at some point f'(a)) and using a slider bar to create the function f'(x); solving equations, plotting a function defined by an integral and computing Taylor series in a much faster way; the new possibility of animating some optimization problems. For the multiple variables course, we can now work graphically with 2D vectors and plot multiple 3D surfaces; again, the faster CPU allows us to face heavy optimization problems and easily compute multiple integrals. For both calculus courses, the fact that we can now plot different 2D graphs in the same window is a great advantage for teaching and understanding mathematical concepts.

As teachers, we are using this technology in two different and complimentary ways: we use it to introduce and explore mathematical concepts and we also want our students to learn some skills in regards to technology. This is now part of the curriculum and will be presented in the talk; also, we will show some examples (taken from exams or take-home work) where the use of Nspire CAS (or Voyage 200) technology is required. Pencil and paper techniques are still important and students should also learn to validate manual results using these tools and understand unexpected results obtained by CAS systems. We will show examples where the benefit of using this approach is obvious.

The keywords:
TI-Nspire CAS technology, calculator, calculus, optimization, animation

P10: Dynamic Systems on Various Platforms

Josef Böhm
ACDCA and Intl DERIVE & CAS-TI User Group
Austria

The extended abstract:

There are not so many sections of school mathematics which are addressing almost all competences appearing in the various competence models as "Dynamic Systems". DS are part of the Austrian curriculum for Upper Secondary Schools.

Predator-Prey, Population Development, and Supply-Demand are only a small selection of possible problems. They are pretty well known and sometimes overstrained.

We will focus on three dynamic systems from various fields of applications:

- Collapse of an Ecological System
- What is a Brusselator?
- Gumowski-Mira-Attractor

Most of the dynamic systems can be modelled by systems of difference equations or by systems of differential equations. It might be nice to find a solution for one special set of parameters but real investigations for fix points, attractors, etc. can be done visually by changing the parameters in an easy way.

In order to reach our goal - a dynamic presentation - we will show how to use sliders for changing the simulation parameters and/or how to drag the initial points of the models.

We will show how to treat the modelling process with various software tools which are available on our schools and discuss their advantages and disadvantages. We will also demonstrate how they can collaborate. The tools mentioned are

- a professional simulation software which is free for educational purposes
- Computer algebra systems
- Dynamic Geometry
- Spreadsheet

It is the presenter's strong opinion that the teacher can strengthen the flexibility of the students by offering different approaches to the modelling process. It makes a difference transferring the system into a system of difference equations and then proceeding in a spreadsheet or proceeding by developing a program supported by a compute algebra system. We also will take the opportunity to introduce a commercial simulation software together with a rich resource of examples.

The keywords:
Dynamic systems, modelling, collaboration of tools

P11: Dynamic Systems – From Simulation Software to CAS

Josef Böhm
ACDCA and Intl DERIVE & CAS-TI User Group
Austria

The extended abstract:

As extension to my lecture "Dynamic Systems on Various Platforms". The interested participants can experience how to model a dynamic system starting with a professional simulation program and ending with a Computer Algebra tool - with an intermediate stop at a spreadsheet treatment.

We will investigate a not too complicated system modelling the interdependence of Environment Quality and Tourism (based on H. Bossel's SystemZoo).

No experience of any program is necessary.

The tools used are VENSIM (prof. simulation software which is free for academic purposes), MS-Excel, DERIVE and/or TI-NspireCAS and/or GeoGebra.

Parts of the computer treatments will be provided by the presenter.

The keywords:
Dynamic systems, modelling, collaboration of tools

P12: Attracted by (Strange) Attractors

Josef Böhm
ACDCA and Intl DERIVE & CAS-TI User Group
Austria

The extended abstract:

When investigating dynamic systems, fractals and chaotic behaviour it is inevitable to come across "attractors". An attractor is a set towards which a dynamical system evolves over time. This can be one ore more points, a curve, or a kind of fractal structure in the phase space. The latter are called "strange attractors".

Investigating these special sets opens a huge box of surprising shapes which might form a linkage between mathematics and arts.

We will use the excellent mathematical capabilities of DERIVE (and other tools) wrapped in programs to present well known and not so well known attractors which can be found in "CHAOS-Literature", as there are: Lorenz-, Henon-, Roessler-, Ikeda-, Gumowski-Mira-Attractors and others.

The keywords:
Fractals, attractors, chaotic behaviour

P13: Find Your Very Own "Attractive" Attractor

Josef Böhm
ACDCA and Intl DERIVE & CAS-TI User Group
Austria

The extended abstract:

The aim of this workshop is to present possibilities how to find unique attractors based on one and more dimensional quadratic, cubic, quartic, etc. maps.

The activities are based on a publication of Julien C. Sprott. We will introduce the Ljapunov-exponent as a means to find attractive attractors using random numbers for defining the generating systems of difference equations.

We hope that each participant can leave the workshop with the satisfying feeling having discovered a new personal attractor.

Additionally a program will be provided for experimenting with several varieties of the Gumowski-Mira-attractor.

We will use ready made DERIVE functions and programs. Special knowledge of DERIVE is not necessary.

The keywords:
Attractors, Ljapunov exponent, more dimensional maps

P14: Learning Mathematics through the Surrounding World

Eva Chepurko
Valga Gymnasium
Estonia

The extended abstract:

Students, usually not interested in math but using ICT and images from the surrounding world, are able to win competitions and start an International project. Is it possible? - Yes, it is.

At the beginning of the speech difficulties of understanding maths, like drawing straight line instead of parabola, will be shown. Also will be share experience of teaching students in a small group.

The speech will give an overview of the possibility to teach mathematics using ICT and the program GeoGebra. The idea is to teach the student to see mathematics in their everyday life and in the surrounding world, to take photos and use them to improve mathematical knowledge. The speech introduces the possibility to use practical settings in studying topics like „Parabola (quadratic function)", „Symmetry" , „Three-dimensional shapes" and " Golden Ratio". Some video material from class will be shown. Also some ideas for complementing regular classroom activities with lessons outside school will be presented. Speech explains how this method has supported the students to start the International project „Math is everywhere‟ http://etwinmath.weebly.com/ involving 8 countries. At the end of the speech will be shown how those students have fewer difficulties in understanding math by carrying out this project and how students get to love Math!

The keywords:
Difficulties in understanding maths, GeoGebra, Maths is everywhere

P15: Representations of Solids and Surfaces within the TI N'Spire Environment

Jean-Jacques Dahan
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France

The extended abstract:

Everybody is aware that using technology enhances a more experimental approach of mathematics: as Thurston and Wendelin (Fields medals), we think that maths are basically experimental. We will illustrate it within the TI N'Spire environment with examples related to the basic concept of coordinates in 3D geometry. We will show how 3D coordinates can be represented in parallel perspective (and especially in military perspective) in the "Geometry" and "Graphs" applications of TI N'Spire. A dynamic approach is possible thanks to the sliders that can enhance a deep understanding in these representations (according to Duval). Dynamic numbers can help students to reach the concept of variable (Jackiw) thanks to the possible "linking" between measurements and variable. Therefore, we will play with variables, sliders and loci to create representations of solids such as cylinders, cones and other surfaces (directly or with the new 3D plotter). A more interesting problem will be modelized: the unfolding of cylinders and cones: this problem originally solved with the Cabri environments, Cabri 2 Plus and Cabri 3D is solved here in using the possible connections between algebra and geometry in TI N'Spire following the principle stated by Laborde that "the teaching of mathematics must help students learn how to adequately use various representations and to move between them if needed.". The special language of experimental maths will be used as defined in my research work about the experimental process of discovery in maths mediated by technology: exploration, generative and validative experiments, conjecture, plausibility, experimental proof ?) to help teachers to understand what to teach and what to assess when they practice experimental math supported by technology.

The keywords:
Ti N'Spire, surfaces, military perspective, cavalière perspective, folding, unfolding, cylinder,cone, slider, 3D graphing tool

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P16: Using Dynamic Documents to Enhance Student Understanding

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USA

The extended abstract:

Software documents that allow students to dynamically engage with mathematical ideas can enhance student understanding of standard mathematical topics. The presenters will demonstrate the use of two such documents along with the questions that illicit student reflection and inquiry. The first document will address the notion of eigenvectors, eigenvalues, and the characteristic polynomial in two-dimensions. The second presentation will address the intricacies of limits of real valued functions in R^2.

The keywords:
Applets, inquiry-based, questioning, linear algebra, limits, real analysis, CAS

P17: Compund Damped Pendulum: an Energy Comparison

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USA, South Africa

The extended abstract:

We use an energy approach (kinetic plus potential) to investigate damped pendulum models. Three models are compared: viscous damped, quadratic damped, and a combination of both dampings which we call compound damping. We will argue that the compound damping model is more realistic than the other two. This argument is based upon deriving velocity equations (and their solutions) that permit expressing the total energy of a solution to an initial value problem as a function of displacement alone, bypassing velocity. A numerical approach using Mathematica (but any good CAS or ode solver could suffice) makes such a determination relatively easy and moreover, maximum displacements from equilibrium of the pendulum for a given set of initial conditions are determined without solving the initial value problem. What makes this mathematically interesting is that there is a jump discontinuity in the quadratic damping term in order that the damping force always opposes the direction of motion.

In doing so, this approach provides a new paradigm for viewing solutions to oscillator models. Rather than considering the entire solution over a long time interval, we consider the solution to be a sequence of half-cycle arcs, rising from a local minimum to a local maximum and then falling from the maximum to a new local minimum. Each half-cycle carries its own energy and our approach quantifies this energy. The technique is suitable for classroom discussion and computer laboratory investigation.

The keywords:
Viscous damping, quadratic damping, compound damping, kinetic and potential energy, pendulum equation

P18: From Vienna to Tartu: A 10 Years Tour using Derive in TIME

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Spain

The extended abstract:

In recent years a strong movement has been taking place among math teachers that use CAS (Computer Algebra System) to change the traditional uses given to these tools. Many teachers believe that it is a mistake to use CAS in teaching as simple problem-solving machines and their use should not be reduced to high-performance calculators. On the contrary, they should be used in ways that maximize the opportunities that these technologies offer, focusing on positively affecting student learning, significantly increasing opportunities for experimentation and allowing students to construct their own mathematical knowledge under the guidance of their teachers. We agree with teachers who argue that the use of CAS in Mathematics has not yet reached optimum conditions. In fact, the most commonly used CAS are black boxes (showing the result in one step without teaching students how to get there), while they should be white boxes (showing intermediate steps).

After a short review about the different uses given to CAS, we will focus the talk in our experience in using CAS, specifically Derive, together with programming. When students program, they must read, construct and refine strategies, modify previously written programs and lastly, use the programs to solve problems. This makes them the protagonists of their own learning. The appropriate use of programming with a CAS will lead us to state the equation:

Power of CAS + Language Program Flexibility = Mathematical creativity

We will describe our different utility files presented in TIME's conferences since these series of Conferences started in 2002 in Wien. These utilities files cover different Math subjects for Engineering such as: Parametrization of Curves, Line Integration, Multiple Integration, Surface Integration, Complex Analysis, Random Distribution Generation, Automated Theorem Proving, Laplace Transform, Ordinary differential Equations and Partial Differential Equations.

The use of these utilities files together with other multimedia resources, have been used in our teaching in Math courses for Engineering in order to use a blended learning (b-learning) methodology with our students. We will also describe how this methodology has allowed us to adapt to the new Bologna Process requirements that it is nowadays implanted in Europe.

P19: Changing Assessment Methods: New Rules, New Roles

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Spain

The extended abstract:

The European Higher Education Area implies a deep change in the teaching of Mathematics in engineering schools. It is no longer possible, for example, a drastic division between lectures with examples, practical classes without a computer and laboratory sessions. Nor is it possible to maintain the same traditional assessment forms.

In the case of use of Computer Algebra Systems (CAS) over the past 20 years, its limitation to the math labs has led to a narrow view by the student: the use of a CAS is an additional work, not included in the learning process.

In this paper, we present two experiments carried out in the academic year 2011-12. Both experiences contribute to establishing a new role in the integrated use of CAS in teaching mathematics. The students can use the CAS throughout all the learning process, including assessment activities.

The first experiment refers to a Linear Algebra course in the degree in Mechanical Engineering from the Polytechnic University of Madrid (UPM). In this course the software chosen was Maxima because it is a freely available and powerful open source CAS easy to use. The second experience was carried out in the course of Mathematical Methods for Signal Processing in the Computing Engineering degree at UPM. The software used was MATLAB, which is the most widely used technical computing software for engineers.

Results including the similarities and differences of experiences will be presented. As well as the analysis of the students' impressions, according with the realized enquiries.

The keywords:
Computer Algebra Systems, assessment's methods, use of technology, team work, engineering mathematics

P20: Adapting the Assessment to the Instrumental Genesis

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Finland

The extended abstract:

Background

The development of information and communication technology together with its usage can be considered as instrumental genesis, triggered by instrumentation and instrumentalisation. The first one means that a person can use the instrument, whereas the latter means that the instrument also shapes the actions and the character of the knowledge constructed with the tool. Whilst students are in their free time often on the level of instrumentalisation when using their personal devices, educational institutions keep on staying on the level of instrumentation putting attention on what should be included in the curriculum, and in which range to allow the usage of technology especially in the assessment. The assessment is still based on quite stereotypic tasks, which very often are also a complement of the problems that exist in real life.

Aims and methods

The article shows why conventional task types cause a deadlock in the assessment. It represents prototypes for new kind of tasks, which can be used for assessment independently of whether they are solved by using technology or not. The analysis is a meta-study based on theoretical considerations on the relation of conceptual and procedural knowledge. This is supported by author?s empirical findings within the so-called ClassPad projects, for example.

Results

The article represents a systematic analysis of assessment tasks, showing 81 potential task types. These types are discussed within the following "trichotomy", still used to administrate examinations in man countries: (i) the usage of technology is not at all allowed, (ii) the technology may be used freely, or (iii) the technology must be used. The analysis reveals the main reason for the current deadlock and suggests drastic shift to new kind of assessment tasks. Research-based prototypes for those tasks are given.

Conclusions

The results might give encouragement for mathematics educators to develop the assessment culture systematically, trying to respond to the well-known challenge that "assessment is the battlefield where the war of improving the quality of mathematics teaching will be won or lost". As in many European countries the usage of technology is allowed in matriculation examination, for example, the potential impacts can be considered remarkable for mathematics teaching in as well schools and universities. One of the conclusions can be expressed as "Instead of speaking about "implementing modern technology into classroom" it might be more appropriate to speak about "adapting mathematics teaching to the needs of information technology in modern society."

The keywords:
Assessment, instrumentation, instrumentalisation, instrumental genesis, technology-based

P21: Systems of Computer Algebra and Dynamic Geometry as Tools of Mathematical Investigation

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The extended abstract:

Computer algebra systems together with dynamic geometry systems provide a user with a means of investigation of the mathematical background of various real world phenomena. The combination of symbolic capabilities with simply-made dynamic models gives us a means of introducing students to, what are in fact, complex problems.

We will present selected examples of the modeling of such problems all based on the joint use of the computer algebra system wxMaxima and tools of the geometry and algebraic package GeoGebra. Particularly we will take advantage of its graphic functions and its built-in spreadsheet, which has appeared as an effective means for an iterative numerical computation. The presented problems belong to the field of calculus and differential geometry. They are applied within the in-service mathematics teacher training programs but thanks to the help of the computer they can be usefully solved at the level of secondary school mathematics and play the role of windows into the higher levels of mathematics and its application.

The keywords:
Mathematical investigation, mathematics teaching, computer algebra system, dynamic geometry

P22: Experiences of 15 Years Using Grafic Calculators in Saxonia

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Germany

The extended abstract:

In Saxony, one state of the Federal Republic of Germany, the use of graphic Calculators is obligatory from the 8th form on the gymnasium (high school). It is also necessary in the central school leaving examination.

To chance the traditional "picture of mathematics" we defined in our new curricula the following general goals of teaching mathematics:

- Advancement of problem solving competence
- Critical use of reason
- Advancement of the competence of using mathematical language appropriately
- Advancement of the ability to visualise things
- Advancement of the competence of using basic mathematical objects appropriately

In ower point of view we see methodical and didactical reasons for the use of technology to support this goals, f.e.:

- explorative learning - experimentation,
- visualisation,
- motivation,
- change of assignment culture.

In the lesson should be shown examples for the use of technology in the lessons and in examinations.

The keywords:
Grafic calculators, classroom examples

P23: Visualizing and Understanding Statistical Concepts with TI-Nspire

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Belgium

The extended abstract:

TI-Nspire is a powerful tool to introduce statistical terms and thinking in an introductory statistics course.

This article discusses a possible approach, using dynamic simulations of simple random sampling and graphical representations, to help students grasping statistical key concepts and enhancing their statistical reasoning.

The examples are suitable for use in a first year college course.

The keywords:
Simulations, probability, law of large numbers, univariate distributions, sampling distributions, central limit theorem, statistical inference

P24: CAS or not CAS? - That is the Question when Using Technology in Mathematics Education

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Austria

The extended abstract:

Many countries define standards for the use of technology in mathematics education, whereby some allow and demand graphic calculators while in other countries CAS technology is the minimum standard. The former argue that the larger part of interesting examples for the use of technology can also be solved with graphic calculators or spreadsheets.

Based on goals of mathematics education I will compare and contrast reasons for the necessity of the use of CAS. I will support my thesis with suitable examples. According to this investigation I will distinguish 5 categories:

C1. Traditional examples where neither graphic calculators nor CAS are helpful
C2. Traditional examples (developed for scientific calculators) which are solved faster or even trivialized by graphic calculators or spreadsheets
C3. Traditional examples (developed for scientific calculators) which are solved faster or even trivialized by CAS
C4. Examples which only can be solved by the use of graphic calculators or spreadsheets (and also CAS)
C5. Examples which only can be solved by the use of CAS

Such as the enormous field of solving problems by using difference equations can be also solved by graphic calculators, spreadsheets a.s.o (category C4) while the limit of productsums on the way to the concept of the integral needs CAS for exact solutions (category 5).

The keywords:
CAS, graphic calculators, experiemntal phase, exactifying phase, application phase, learning process

P25: Using Fourier Series to Analyse Mass Imperfections in Vibratory Gyroscopes

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South Africa

The extended abstract:

Standing waves can exist as stable vibrating patterns in perfect structures, such as a solid cylindrical body. Any manufacturing imperfections in the body destroy the standing waves. Resonator gyroscopes cannot be manufactured without imperfections (anisotropies) and these imperfections cannot be ignored because they cause departures from ideal mass, stiffness and damping distributions and therefore a¤ect resonator dynamics. Using a CAS in order to illustrate this to undergraduates with a working knowledge of college calculus, we consider the vibrating pattern of a slowly rotating solid cylindrical shell where a slight mass imperfection is introduced via a Fourier series into the equations of motion of the vibrating particles in the shell. We demonstrate that the mass anisotropy induces a frequency splitting (beats). A special system of four time dependent variables is introduced that allows us to study the amplitudes of the principal and quadrature vibrating patterns as well as a phase shift and the precession angle associated with Bryans e¤ect. A system of four coupled first order nonlinear ODE is produced that is not easy to interpret. However, using the method of averaging, a system of four coupled first order nonlinear ODE is produced that is both easy to analyse and that models the four variables to a good approximation initially, showing similar trends to that of the non-averaged system globally. It appears that the inertial angular rate does not influence changes with time in the amplitudes of the principal and quadrature vibrating patterns or the phase shift. The mass anisotropy cause changes with time in all four variables. If a mass-anisotropy is present then a capture effect occurs with the precession angle that appears to vary periodically and not increase linearly as it would for a perfect structure. Using the CAS Mathematica to do the analysis involved renders this work accessible to the above mentioned undergraduate students.

The keywords:
CAS, senior undergraduates, Fourier series, resonator gyroscopes, frequency splitting, Bryan's effect, rotating shell, mass anisotropy, nonlinear ODE

P26: Critical Issues of Effective CAS Utilization

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Serbia

The extended abstract:

This presentation examines several critical issues of effective CAS utilization. By using a number of examples from an upper secondary mathematics education, these issues deal with CAS intrumentalization and CAS-supported assessment design. Regarding this instrumentalization, the presentation considers the extent to which the instrumentalization can be done with CAS tools at present, and discusses several critical issues in doing so with respect to tool, task and designer (learner). Concerning this assessment design, the presentation calls for a design that gives explicit suggestions about solving tasks by CAS (e.g., what tasks may be solved by CAS but with a care; what kind of CAS-based solutions are appropriate), as well as provides some direct scaffolding in using CAS to enable students be more concerned about an interplay between mathematical issues and CAS affords. Implications for research and practice are included.

P27: Types of Mistakes and Dynamics of the Occurrence of Mistakes Solving Linear Equations Using Computer Program T-algebra

Janika Kaljula
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Estonia

The extended abstract:

T-algebra is an interactive learning environment for exercises in four areas of school algebra: calculation of the values of numerical expressions; operations with fractions; solving of linear equations, inequalities and linear equation systems; operations with monomials and polynomials. The student solves the task step by step. At each solution step the student selects the operation, indicates the subexpression(s) to be changed and enters the result of conversion. The program checks all three stages of the step, gives feedback and hints. The program records the information about the solution process and mistakes.

This proposal is lying on a report on findings of the study, exploring the types of mistakes solving linear equations and how these mistakes change during the learning process. The study was carried out on linear equations, which were studied by 110 of the 7th grade students. The data were gathered as computer files, which were created during the practicing lessons solving linear equations (5 lessons) using computer program T-algebra. Statistical procedures included t-test and nonparametric tests. The most frequently appearing mistakes are sign errors: minus sign before fraction is not taken into account, minus sign before parentheses is not taken into account; calculation errors: arithmetic mistakes in dividing, multiplying, in combining liketerms and in adding/substracting a number. Often were taken errors where rule doesn't correspond to the solution algoritm and selected syntactically incorrect object.

Results of this study allow giving advice to the authors of textbooks and improving learning in a regular class.

The keywords:
Linear equations, T-algebra

P28: High School Students' Acquisition of Knowledge and Skills through Self-Organization and Collaboration

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Estonia

The extended abstract:

Even if we realize that we live in a rapidly and ever changing world and understand that the impact of technology on our lives is increasing dramatically; even if we know a 21st century competencies and that a 21st century teacher is: an adaptor, a communicator, a learner, a visionary and a model, a collaborator, a leader and risk taker; even if we have a 21st century technology in the classroom and different web-based instruments to use - it is not enough to be a 21st century teacher. We will be a 21st century teacher only if how we teach changes as well.

Our approach to teaching must change, this means that we have to (A. Churches):

- focus on the move from lower order thinking skills (industrial age of education) to higher order thinking skills;
- teach contextually and interdisciplinary;
- work collaboratively;
- use project-based learning;
- foster problem solving;
- build technological, information and media fluencies;
- assess transparently.

In our presentation we will also discuss three models of teaching: "Direct" (Teacher-Delivered), "Radical" (Student-Centered), "Social". Especially we will focus on the blend of "Radical" and "Social" models of teaching. During presentation we will present different cases of how we already changed our teaching in grades 10 - 12, how technology and web-based instruments helped us to realize our approach of teaching mathematics in practise. Also we conducted different feedback tests for students and we are planning to discuss these results and our future plans with participants of our lecture.

The keywords:
21st century teacher, collaboration, teaching models, Bloom`s taxonomy, assessment

P29: Using Derive To Generate DNA Fractal Representations

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USA

The extended abstract:

The presentation will introduce the use of DERIVE for the generation of fractal images of DNA sequences. It is based on the work of H. Joel Jeffrey in his 1990 paper, Chaos game representation of gene structure. Starting with the random generation of the Sierpinski Triangle, Jeffrey noticed that the outcome wasindependent of the quality of the random number generator. Applying the Sierpinski algorithm to a four sided figure produced a square filled uniformly with dots. Jeffrey decided to try, instead of a random sequence of points, a sequence generated by the DNA encoding of a gene. He reasond that since the structure of the sequence should produce a structured pattern. In this presentation we will follow Jeffrey's investigation. Indeed, we will show the fractal nature of the figure that is generated. Along the way we will introduce some basic ideas about DNA and genes.

Since the goal of this presentation is to illustrate a project that can be done by advanced secondary students and lower level university students, it is designed so that teachers can adopt it as classroom project. Thus, they can introduce their students to a marriage of three interesting subjects: mathematics, bioinformatics and computer programming.

The role of the CAS in this presentation is that all points are calculated using fractional arithmetic giving a more accurate placement of points within the fractal pattern.

The keywords:
Derive programming, bioinformatics, DNA, fractal

P30: Hidden Markov Models and an Introduction to Their Uses in Bioinformatics

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USA

The extended abstract:

Following the agenda found in Chapters 3,4,and 5 of "Giological Sequence Analysis" by Durbin, Eddy, Krough, and Mitchison, (Cambridge University Press 1998, this presentation will introduce the ideas of Hidden Markov Models and some of their uses in Bioinformatics. The initial introduction of HMM's will be done using the book's example of the "occasionally dishonest casino" intended to present HMM's in a more easily accessable setting. Following the introduction of the basic concept some of the fundamental algorithms related to HMMs. In particular, the Viterb, forward, backward, and Baum-Welch algorithms and their implementation in DERIVE will be presented. The presentation will illustrate some of the uses of these algorithms for nucleotide DNA sequences.

The goal of this presentation is to make some of the funamental ideas of bioinformatics and uses of HMM's accessible to advanced secondary school and beginning university mathematics and science students.

The keywords:
Hidden Markov models, bioinformatics, algorithm, DNA sequences

P31: Finding Bioinformatics Data and DERIVE Programs to Process It

L. Carl Leinbach
Gettysburg College (retired)
USA

The extended abstract:

Bioinformatics is an exciting area for processing molecular biology information. It is rich area for applications of mathematics and computer science. In this workshop the participant will explore some of the many rich areas for obtaining data and also algorithms for analysing and processing the data. While most algorithms require large scale algorithms, the writing of DERIVE programs to process smalldata sets will give introductory students insight to the inner workings of these algorithms. Students will also gain a perspective on the mathematics undelying popular detective fiction, television programs, and movies.

The presenter will also share his experiences team teaching an introductory bioinformatics course with a microbiologist that they designed for introductory biology, computer xcience, and mathematics students.

The keywords:
Bioinformatics, algorithms, molecular biology, data

P32: Being a Math Teacher - Looking through the Blog Posts of Student Teachers

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Estonia

The extended abstract:

Blogs represent one kind of social software that is increasingly employed to enhance communication environments in the educational domain (e.g. Curchill, 2011; Hourigan & Murray, 2010). Researchers have outlined blogs as tools for promoting reflection (e.g. Richardson, 2006; Xie, Ke, & Sharma, 2008) and develop learning community (e.g. Snyder, 2009; Sun, 2010). In teacher education blogs are used to measure student teachers' professional development (Chuang, 2010). Because Wopereis, Sloep, & Poortman (2010) claim that blogs externalize thinking process blog was created to externalizing how student teachers reflect their experiences during teaching practice, how they feel being beginning math teachers.

Nine student teachers entered to teaching practice in 2011. They were in six separate schools in Tartu. Because the ability to share feelings and thoughts is important for supporting student teachers (Capa Aydin & Woolfolk Hoy, 2005) we created to them cooperative blog (blogspot.com) for sharing ideas. There were no specific tasks to complete on the blog – they were simply asked to use the blog voluntarily to write about topics they wanted to share with the others or to ask questions of the others. One of the supervisors participated in the blog as well. The importance and necessity of the instructor participating in the blog has been discussed in several papers (Top et al., 2010; Yang, 2009). The blog was closed and only the participants of the study and the supervisor could read and write on this blog.

133 new posts and 242 comments from students were posted. The period of practice was divided into 5 sub-periods. Quantitative content analysis was used for data analysis of the blog posts. In presentation is discussed about what and how they wrote in blogs during the teaching practice.

P33: Mathematical Induction and CAS

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Germany

The extended abstract:

First we give some classical motivational examples for the neccesary of mathematical induction.

- a) The term x^2+x+41 gives for x = 0, 1, …, 39 primes (L. Euler). Moreover the term x^2-79x+1601 leads for x = 0, 1, …, 79 to primes.
- b) If we factorize the polynomial x^n-1 we only (?) get the coefficients 1, 0 or +1 (N. G. Tschebotarew, 1938)
- c) Is there a positiv integer n so that 991n^2+1 is a square?

The use of CAS for the proof with mathematical induction is discussed. Finally, an inequality will be proved by mathematical induction and we show that the corresponding weaker inequality cannot be proved by induction. The reasons of this paradox are explained.

The keywords:
Mathematical induction, CAS

P34: Finnish Upper Secondary Students' Collaborative Processes in Learning Statistics in CSCL Environment

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Finland

The extended abstract:

This design-based research focuses on to document statistical learning among 16-17 year-old Finnish upper secondary school students? (N = 78) in Computer Supported Collaborative Learning environment. One novelty value of this study is in reporting the shift from teacher-led mathematical teaching into autonomous small-group learning in statistics. The main aim in this study is to examine: How students' collaboration occurs in learning statistics in CSCL environment?

The research-data includes material from pre-tests, students' feedback from teaching experiment, video-taped classroom observations and interviews, course exam, delayed post-test, and the notes of the researcher. In teaching experiment students were able to exploit Internet, interactive pdf-material and computer programs in producing assignment of given statistical problem in CSCL environment.

In this paper the intersubjective phenomena of students interaction in CSCL environment is analysed by using Contact Summary Sheet. Aptly selected episodes were transcribed and coded into functional roles, level of collaboration and salient signs of student's group cognition by using the CSS-instrument. Basic unit in data analysis is the small-group-event in one statistical task. In the video-content analyses students' interactions or utterances were divided into not-task-related and task-related categories to point out student's level of collaboration. The reliability is gained by expressing the agreement between two independent coders.

Based on tentative results students' reactions to CSCL were inspired. Collaborative learning can facilitate cohesion and responsibility and reduce students' feelings of detachment in our classless, periodic school-system. Participated students were able to use the interactive material asynchronously, in their own speed, for instance to repeat or pause on the theoretical clips within the interactive material if there was something unclear. This provided an opportunity to discuss and collaborate in small-groups and enabled small-group knowledge-building.

The results indicate that collaboration benefits students' statistical problem solving and collaboration is more intrinsic to girls. It was obvious that the students need more rehearsal in learning mathematics collaboratively to improve personal competence of collaborative learning. Students as well as teachers were more used to teacher-led mathematic teaching than utilizing new methods in mathematics teaching. CSCL is one possible method to foster and facilitate statistical teaching.

The keywords:
Statistics education research, computer supported collaborative learning, design-based research, collaborative learning

P35: Making Learning Math Addictive - Localization of the Famous Open Source Website Khan Academy in Estonia

Robert Peetsalu
NPO KAE Kool
Estonia

The extended abstract:

I'm going to talk about my volunteer project aiming to create a localized Estonian analogue to the famous e-learning platform Khan Academy (www.khanacademy.org), about what's so great about the Khan Academy (besides videos) and about how potent is the idea to create a national analogue to Khan Academy.

In my presentation I'm going to give an overview of how I'm finding new authors of tutorial videos, how I'm using crowdsourcing to translate Khan Academy videos into Estonian and how I'm organizing volunteers to develop our new website on the basis of Khan Academy source code. The result will likely be a precious resource for self-directed and self-paced learning in the students' mother tongue.

In my lecture I will also point out the benefits of Khan Academy for individual learning as well as it's potential use in educational institutions.

The keywords:
E-learning, self-paced, gamified, Khan Academy

P36: Programming-like Activities for Calculus and Differential Equations Classes

Dennis Pence
Western Michigan University
USA

The extended abstract:

CAS computer applications and the handheld devices used in mathematics instruction offer several ways to do what I call "programming-like activities." In English, the word "programming" has come to be used almost exclusively for "computer programming." [However we talk about "television programming," and we are handed a "program" when we attend a conference, indicating a broader meaning for the word "programming".] Many students and instructors have a very negative attitude about writing computer programs or calculator programs. But we do teach certain algorithms, and we do study iterative methods that involve step-by-step methods. Programming-like activities require that students do the following: (1) plan and organize the implementation of an algorithm or iterative process, (2) carry out the steps efficiently and accurately, and, sometimes, (3) decide when to terminate the process. The logical reasoning needed to complete these steps, in the correct way and in the proper order, is very much a mathematical activity that deserves attention in our courses.

Newton's method is one good example. This is an iterative process that (when it converges) gives results closer and closer to a desired zero of a function. All too many textbook questions on this topic give a starting value and specify a small number of steps, virtually eliminating all of the aspects that I would call programming-like. Of course this gives an exact answer for the textbook. But a plan for the whole process needs to determine an appropriate starting value. Since we know Newton's method can diverge, the plan must consider when to terminate with failure or when to stop if we are close enough.

Numerical methods for solving differential equations, such as the Runge-Kutta method, are taught, even in a first course on differential equations. Textbook questions tend to specify the step size and require a small number of steps, again taking all of the decisions and planning out the task. Here is a variation that takes more planning. Consider numerically solving an initial-value ordinary differential equation for the purpose of determining where the solution first takes on the value zero. Thus you do not know ahead how many steps will be needed in the iteration. Instead, the iteration will continue until the solution changes sign from one step to the next. [An alternate termination after a fixed number of steps might report "failure" for a solution that is never zero.] Since the zero occurred somewhere in the middle of the last successful step, you need to interpolate and more accurately estimate where the first zero occurred.

All of this decision-making and planning will require a much greater understanding of the algorithm or iterative process that is used as a tool in the programming-like activity. Simply running a computer or calculator program written by others or performing a small number of hand computations to get a numerical value (printed in the back of the book or checked by the instructor) does not usually lead to much understanding about what is happening within the numerical method or algorithm of interest.

The keywords:
Programming, algorithms, numerical methods

P37: Notice Mathematics around You

Sirje Pihlap, Sirje Sild, Kristi Kreutzberg
University of Tartu, Nõo Upper Secondary School, Tartu Veeriku School
Estonia

The extended abstract:

Estonian students have achieved good results in international comparative assessments of TIMMS 2003, PISA 2006 and 2009 (Lepmann 2006; 2011). However, those assessments have enabled to identify certain problems that require closer attention: our students do not like to learn mathematics and they do not regard it as a valuable subject in terms of their future plans. At the same time, Estonia, as other countries of the EU, needs to increase the number of graduates in mathematics. According to several studies (Pihlap, 2006; 2010; Luik 2004), learning mathematics can be made more interesting, rewarding and fun by the use of computers. Several software applications, including Wiris computer algebra software and GeoGebra dynamic geometry software, are available for mathematics instruction in Estonia. The programs have been translated into Estonian and teachers have an opportunity to learn to use the software in free training. However, according to Prei (2010), 60% of mathematics teachers have never used these programs. A student competition "Notice Mathematics around You" has been organised in Estonia for five years to promote the use of the aforementioned software applications and to emphasise connections between mathematics and daily life. In two years, the contestants have prepared text problems, whereas the initial data had to be found from everyday life in the first competition and from various school subjects in the second. The participants then had to use the prepared problem to compile an equation or equation system, and to solve it in Wiris. In two competitions, the contestants were asked to identify and photograph mathematical objects (lines in one year and three-dimensional shapes in the other) in the surrounding environment, and to mark them on the photographs using GeoGebra. The assignment in the fifth competition was to create patterns from animated lines using GeoGebra software. In all competitions involving GeoGebra, the contestants had to submit mathematical explanations of drawings in addition to a GeoGebra worksheet. In the opinion of teachers, such competitions have been motivating for students. Even students who had never participated in traditional mathematics contests were able to have experiences of success. The students believed that the competitions have provided them with new knowledge and skills along with the joy of discovery. Several contestants stated that were not able to see mathematics everywhere around us before the competition.

Acknowledgement: This abstract was supported by ESF.

The keywords:
ICT, CAS, GeoGebra, Wiris, mathematics

P38: A Tool for Evaluating Solution Economy of Algebraic Transformations

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The extended abstract:

At our department, students solve exercises of the third-term course in Mathematical Logic on computers since 1991. In this paper we consider exercises on finding the disjunctive normal form of propositional formulae. For example, the program generates the formula ¬(A~C)&(B⊃¬AvB). The student should express implication and equivalence through negation, conjunction and disjunction, use De Morgan's laws for pushing negations down to variables, expand the formula using distributive law, etc., until getting the full normal form A&B&¬CvA&¬B&¬Cv¬A&B&Cv¬A&¬B&C.

Working in our formula manipulation environment, the student marks for each step a subformula and enters the result of conversion or chooses a conversion rule from the menu (29 rules available). The environment checks syntactical correctness (of marking and of input), equivalence between marked and entered subformulae, and applicability of selected rule. In case of an error the program issues a corresponding message and requires correction. As a result, recorded final solutions do not contain any direct mistakes. However, the environment does not check solution economy.

Some years ago the introductory part of propositional logic was moved into the first-term course in Elements of Discrete Mathematics (to improve students' preparation for database and programming courses). We suddenly experienced that the solution files of many students appeared to be 3-4 or more times longer than normal. This was particularly noticeable in normal form exercises, which were solved in a rule-based working mode. An additional tool of solution analysis was implemented for fast identification of straying points in bulky solutions and for collection of statistics. The tool scans the solution and identifies the steps that do not correspond to the normal form algorithm. Inappropriate steps are classified by their nature into 15 types. Step displays and collected statistics are recorded in a text file for further observation. The tool also creates a table with statistics of the entire student group.

The presentation demonstrates our main environment and the analysis tool. We discuss the results of the final test of the course in Elements of Discrete Mathematics (162 participants, 132 completed and 30 partial solutions, 1249 suspicious steps diagnosed, the longest completed solution containing 269 steps). We also draw some conclusions for instruction and about functionalities that can be added to the main environment.

The keywords:
Algebraic transformations, normal forms, rationality of solution

P39: What was Special in DERIVE?

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The extended abstract:

Undoubtedly, all kind of software evolves at a very high speed. Moreover, meanwhile in the past, the launch of new versions only followed drastic changes, nowadays many pieces of software systematically present a new version every year, that supersedes the previous one.

In markets different from the software one, not being able to anticipate and guess the new tendencies can produce a decline in sales. In the software market, some sales leaders have followed a quick <<extinction>> for these same reasons. For instance, regarding word processors, the <<de facto standard>> WordStar was substituted by WordPerfect, itself replaced by MS-Word.

Regarding the computer algebra systems (CAS), the decline of the commercial pioneers of the 60s, Macsyma (still alive as Maxima) and Reduce, would deserve an in-depth study. Even if they evolved in parallel with the nowadays leaders Mathematica and Maple, they lost their leadership. Maxima and Reduce can now be obtained free of charge.

Sometimes, the lack of a critical mass of users can make a powerful piece of software to finally fall into abandonment. That is what probably happened with the revolutionary CAS Axiom, where the algebraic structure where the computations had to be performed was fixed by the user.

But, what has happened with DERIVE after being abandoned by TI, when it was very popular (if not at the peak of its popularity)? Has it followed a step decline? It doesn't seem to be forgotten by users, if we compare bibliographic cites in a search engine such as Google Academic.

Even if programming in DERIVE wasn't as simple or comfortable as in other CAS, still many math teachers that used it in the past consider it the simplest computer algebra system ever for beginners and the more adequate for secondary / college education. This is probably because it was <<only>> used as a (powerful) algebraic calculator.

Some of DERIVE's advantages make sense no longer (for instance, the DOS version of DERIVE could run in PCs without a hard-disk), but others like it icons-oriented menus are still remembered. There were and are different approaches trying to simplify the use of computer algebra systems following this line, like that of the first versions of VXMaxima, with a front end close to DERIVE's, or Maple's palettes. The absence of statements separators (as a single entry line was used) and the by default use of uppercases and lowercases are sometimes underlined as advantages of DERIVE.

Another line of development is language flexibility: for instance, the astounding CAS Xcas allows four different syntaxes: its own, Maple's, MuPAD's and that of TI-89/92's calculators (it also incorporates a simple dynamic geometry system plus turtle graphics).

So, is it only nostalgia or is there something special about DERIVE? The authors, with a long experience in teaching using DERIVE and other computer algebra systems will give their opinion about these questions.

The keywords:
Computer algebra systems, GUI, teaching maths with technology

P40: Teaching Conics through Mathematical Modelling

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The extended abstract:

In this workshop we are going to illustrate the use of The Geometer's Sketchpad (v5) in Teaching Activities about the study of circumference and ellipse. The workshop will be developed in the context of the teaching model Learning Mathematics, Doing Mathematics (LMDM, Flores, 2007, 2010), in which we engage students in Mathematical Modelling problems. The main goal in these kind of activities is not to teach modelling, but use modelling as teaching and learning strategy.

In LMDM we classify modelling problems into two types: curve fitting problems, and think-and-act problems. The first kind are those in which we have a set of data than can be put into a cloud graph or into points in a Cartesian reference system, and we have the task of fitting a curve into those points in order to get a model of the phenomenon we are studying (depending on the academic level and matter subject we can use different methods of fitting as minimal squares or just take some points and try to find the functions that, according to the student, is the best fitting function). The second type are those problems in which you can tell in advance what kind of model is appropriate in order to explain the phenomenon, you only need to adapt the parameters of your model in order to fit the conditions of your specific situation. We also can have combinations of these two kinds of problems.

In the workshop we are going to solve two modelling problems with the aid of the DG software and reflect on the mathematics that involve each kind of problem. Of particular interest is the curve-fitting problem in which we are going to compare the mathematics needed in finding an ellipse equation by analytical methods, and the one needed in using the dynamic features of the software.

The activities are part of a research project developed with funds of Infocab (PB10011) of the National Autonomous University of Mexico (UNAM).

The keywords:
Mathematical modelling, Dynamical Geometry Software, learning mathematics, doing mathematics

P41: An Evaluation of Students' Experiences in Technology-based Courses in Mathematics and Statistics

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The extended abstract:

At the Schmalkalden University Faculty of Business and Economics, two of the four compulsory courses in mathematics and statistics are no longer held in a traditional classroom setting (using blackboard, overhead projector, and pocket calculators) but in the PC lab (one or two students in front of each PC, teacher's PC connected to a projector), where a Computer Algebra System (CAS) and a widely-used statistical software package are continuously accessible. The two mathematics courses are "Introductory Mathematics and Calculus" (in traditional classroom) and "Matrix Algebra and Decision Analysis" (matrix algebra portion in PC lab), the two statistics courses are "Introductory Statistics" (in traditional classroom) and "Computer-assisted Statistics" (in PC lab).

The faculty's CAS license also covers the private PCs of the students such that they can use the software at no charge at home as long as they are enrolled at our faculty. Students also have access to the CAS and the statistical software package during the final exam in the PC lab (here, naturally, only one student per PC).

While the courses and the examinations have been taking place in the PC lab for a number of years, throughout the academic year 2010/11 a series of four surveys was carried out for the first time to find out if the students prefer traditional or technology-based courses, and how well they cope with the technology. Two surveys took place with the 2009 cohort of students at the beginning and at the end of the "Computer-Assisted Statistics" course; the remaining two surveys took place with the 2010 cohort of students in the middle of the "Matrix Algebra" course and at the beginning of the "Computer-Assisted Statistics" course. Results from the first survey (October 2010) were presented at last year's ACA conference. Some interesting results, including comparisons between the two cohorts, and within each cohort over time, will be presented.

The keywords:
Technology-based teaching, matrix algebra, statistics, Computer Algebra System, survey

P42: Deriving Big Formulas with Derive and What Happened Then

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The extended abstract:

I participated in the famous first Krems conference in 1992. At that time, twenty years ago I was, and still am, very fascinated by the fact that it had become possible to produce big formulas in a very comfortable way using a CAS system like Derive.

In Krems I gave a lecture where I demonstrated how to produce big formulas dealing with the circumcenter and incenter of a triangle. I also demonstrated how to use these formulas in other pieces of software like, for example, Excel. I will perform such a demonstration also at this conference.

I will also also deal with incenter and excenters of a tetrahedron.

Now and then I have returned to the above questions. I have found that the expression for the coordinates of the incenter of a triangle can be regarded as the intersection point of any three concurrent lines passing the vertices of a triangle. A consequence of this was that I found a simple necessary and sufficient condition for three lines passing through the vertices of a triangle being concurrent.

I will use the above condition to give very simple proofs that the altitudes, angle bisectors and medians of a triangle are concurrent.

I will also use the DERIVEs wonderful ITERATES-function to visualise and indicate a proof that the medians of a triangle are concurrent.

The keywords:
Incenter, excenter, concurrent lines, triangle, tetrahedron, Derive

P43: Modeling the Probability of a Binary Outcome

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The extended abstract:

Now we are seeing a growing interest in machine learning as an integral part of the discipline of artificial intelligence. Some of the ideas of machine learning are discussed in the course of artificial intelligence for students of computer science. Among the topics covered in this discipline, the methods of regression analysis have a special place. The meaning of the regression analysis can be reduced to finding an analytic expression, the most accurately reflects the relationship between the dependent variable and a set of independent variables.

The study of various aspects of machine learning requires considerable mathematical training. The prerequisites for this course are: linear algebra, nonlinear programming, and probability at a level expected of junior or senior undergraduate in science, engineering or mathematics.

In this paper, the solution of the classification using logistic regression is considered. The main difference from the multiple logistic regression model is the interpretation of the regression equation: logistic regression predicts the probability of the event, which is in the range from 0 to 1. The main reason is the following: students should become more familiar with logistic regression, and start using them in modeling the probability of binary outcomes such as in earth sciences, natural hazard evaluation, medical diagnosis, homeland security, finance, and many others.

Comparison of different methods of solving the problem, including using Excel, Octave, Maple, is given.

The keywords:
Machine learning, logistic regression model, Excel, Octave, Maple

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P44: TI-Nspire CX CAS- New Release 3.2

Gerhard Stolz
Texas Instruments
Germany

The extended abstract:

- Authoring Conditional Attributes
- Chemical Notation - ChemBox
- Color for Math Boxes and Chemical Boxes
- Image, Expression and Chemistry Question Types
- Linear and Conic Graphing
- Offline Mode for TI-Nspire(tm) Document Player
- Press-to-Test Updates
- Publishing Aesthetics - Enhanced Notes
- Science Tools delivered as TI-Nspire(tm) Documents
- Software Installation and Handheld Operating System Updates
- TI-Nspire(tm) Script Editor

P45: Mathematical Modelling with Geometry Expressions and TI nSpire CAS

Philip Todd
Saltire Software
USA

The extended abstract:

Mathematics environments such as Derive or the TI nSpire CAS can be used to help students solve more realistic problems. A Symbolic Geometry environment such as Geometry Expressions can be used to help students model more realistic problems, which can then be solved with the aid of their CAS. Geometry Expressions takes a model which is expressed purely geometrically, and converts it into algebraic expressions which may then be exported to a Computer Algebra System. It can conveniently separate model formulation from model solution, and this can be very useful pedagogically.

In this talk we will present a number of problems which exemplify this approach. Problems will range in mathematical content from elementary geometry, through calculus to differential geometry. We will look at application domains from the placement of irrigation circles in arid west Texas to optimization in the design of solar cookers.

For example, a solar cooker is modeled in Geometry Expressions by a portion of a parabola, the target by a circle centered at the focus. The critical angle (where reflected sunlight misses the target), is computed by the software as an algebraic expression in terms of solar concentration ratio k, and f-number. A maximum for this angle as a function of f-number may be determined by copying the expression into a CAS, differentiating and solving.

The keywords:
Mathematical Modelling, CAS

P46: From Inductive Reasoning to Proof by Induction with Geometry Expressions

Philip Todd
Saltire Software
USA

The extended abstract:

We use Geometry Expressions to calculate the radii of circles in a particular Pappus Chain. We will use inductive reasoning to conjecture a general form for this sequence of radii. We will then use mathematical induction to prove the formula for a general member of the Pappus Chain. Geometry Expressions will be used both in the inductive reasoning component of the problem and (less conventionally) in the proof by induction component. In fact we will create a quite beautiful geometric instance of a proof by induction within the program. We will generalize our result, and observe that the pattern followed by the radii is much more apparent in the general form than in the specific.

This lesson only uses 5 Geometry Expressions commands, and is therefore highly suitable for the beginner user. In particular, the participants will learn the difference between a constraint based approach to creating a model (used by Geometry Expressions) and a construction based approach (used by most other geometry systems). They will also observe the benefits of having symbolic rather than simply numeric output from their geometry environment.

The keywords:
Geometry, induction, CAS

P47: When do Computer Algebra Systems Offer Unexpected Answers to School Equations?

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University of Tartu
Estonia

The extended abstract:

In a large number of cases when solving school equations (from linear and quadratic to trigonometric and literal) with Computer Algebra Systems, the system gives the answer that is expected by the student or teacher. However, this situation occasionally also reveals certain phenomena in which the answer is somewhat different (unexpected). 29 phenomena were identified and grouped in [Tonisson, 2011]. The groups were labeled as Input, Form, Unfinished, Domain, Branches, and Automatic.

The paper for the conference TIME 2012 ascertains more exact bounds between equation subtypes that cause specific types of unexpected answers. In different Computer Algebra Systems the bounds may lie differently. There are five Computer Algebra Systems (Axiom, Maxima, Sage, WIRIS and WolframAlpha) under consideration in the paper. The described scheme is also applicable for other systems and versions.

Sometimes it is quite easy to predict the possible unexpected answer without solving the equation. In many other cases it is necessary to solve the equation for the correct subtyping of the equation. The criteria for subtypes should be understandable and practicable for the teacher. Then the teacher could use it to bring out the educative examples or to avoid the disturbing ones. In the paper, more attention has been paid to didactically more important cases such as those related to number domains or equivalence. These topics are essential but could be somewhat upstaged in school mathematics. The unexpected answers offered by Computer Algebra Systems could bring them into focus.

[Tonisson, 2011] Tonisson, E. (2011). Unexpected answers offered by Computer Algebra Systems to school equations The Electronic Journal on Mathematics and Technology, 5(1), 44-63.

The keywords:
Computer Algebra Systems, unexpected answer, equations

P48: Computers in School Mathematics in Estonia. Possible trends

Eno Tõnisson
University of Tartu

Estonia

The extended abstract:

Computers have been used for learning and teaching mathematics for years. Different countries have different strategies for and experiences of organizing activities on the national level. This discussion focuses on Estonia. We look briefly at the past and outline some possible scenarios for the future. We encourage the audience to bring out possible positive and negative aspects of the scenarios. As Estonia is a small country and sometimes quite flexible, it is possible that the future will be such as we "decide" at the conference.

P49: Integration of Digital Tools into the Mathematics Classroom: A Challenge for Preparing and Supporting the Teacher

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University of Education Niederösterreich
Austria

The extended abstract:

Research findings about the impact of digital technologies on learning of mathematics in schools are rather inconsistent. However, research into the educational effectiveness of technology suggests that, under the right circumstances, it can support and facilitate student learning. On the other hand, teachers do not yet extensively exploit the potential of educational technologies and do not feel appropriately prepared. Although technology is readily available in schools, some teachers do not know how to take advantage of it and still others are against it. Results of investigations suggest that these teachers' resistance is related to their beliefs about mathematics teaching and learning and their existing pedagogies. Teacher may be either uncomfortable with technology, are unsure how to incorporate technology into their curricula, or have not seen examples of effective use. Therefore, the key challenge remains for mathematics teacher educators to design, implement, and evaluate new professional development teacher programs that support the evolution of knowledge, skills, and dispositions for teaching mathematics with technology.

In other words, what do mathematics teachers need to know about educational technology and how they can acquire this knowledge? These questions have guided the design of a teacher training course addressing the development of Technological Pedagogical Content Knowledge (TPCK), as conceptualized by Koehler and Mishra. Accordingly to this aim, the course, which was based on the LoTI framework of Moersch, should support practicing teachers in understanding how the use of particular technologies changes both teaching and learning. The paper reports on various examples, which provide math teachers with opportunities to develop their individual TPCK, needed to incorporate technology in the context of teaching and learning mathematics. Mainly focusing on pedagogical techniques for using technologies in constructive ways to teach mathematics content, the course additionally encouraged teachers to also reflect on the effects of the particular strategies for integrating technology. The study examined how teachers used computer-based technology to enhance their lesson plans, by selecting appropriate technology tools from the course materials and creating learning opportunities for their students.

The keywords:
Technological Pedagogical Content Knowledge, math teacher prepartion and professional development programs, integration of digital tools, math classroom, challenge for teacher educators

P50: Motion Sensors in Mathematics Teaching: Learning Tools for Understanding General Math Concepts?

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Austria

The extended abstract:

On the one hand, physical experiments have a remarkable potential in maths lessons. For instance, findings from case studies suggest that it is possible for nine-year olds to interpret graphs they generate through interactions with sensors.

Furtherore, eleven-year old learners can gain an intuitive understanding of basic calculus concepts by using a position sensor with a computer that generates a real-time graph of the learner's motion and velocity. On the other hand, mathematics has often been called the language of physics. Therefore, incorporating technology tools into the mathematics classroom adds a new dimension to the teaching of math concepts. Especially, gathering data in a hands-on and real time method helps classrooms coming alive and promotes the development of understanding the connections between maths and physics. Researchers exploring educational technologies have many examples of approaches that allow students to learn far more, better, and earlier in contexts that take advantage of the educational impact of information technologies. For example, important concepts of rate and change can be learned even at surprisingly early grades with the motion detector.

By enhancing learning experiences with computer-based motion detectors, six tasks, which were used in study, aimed to support students in making conjunctions between abstract math concepts and fundamental physics phenomena by involving them in mathematical representations of real-world phenomena. Within a physics lab escpecially focusing on the development of maths concepts, students used motion sensors to collect physical data that were graphed in real time and then could be manipulated and analyzed. Thus, due to the fact that data were presented in an immediately understandable graphical form, students were allowed to take an active role in their learning by encouraging them to construct mathematical knowledge from observation of the physical world. Research was conducted with 44 students from grade 9 within three successional years. Data sources included video and audio recordings of students and teacher during three 90-minute sessions, students' written notes, semistructured student interviews and the teacher's journal. For testing students' knowlegde gains the TUG-K test was used within a pre, post and follow-up design.

Findings suggest that, utilizing a predict-observe-explain format, students learned about slope, determining slope, and distance vs. time graphs through motion filled activities. Furthermore, exploring the meaning of slope, viewed as the rate of change motion, students acquired competencies reading, understanding and interpreting kinematics graphs involving a multitude of mathematical representations. As students worked to recognize patterns and made generalizations, graphing tools in combination with motion sensors enabled students to efficiently move among tabular, graphical and symbolic representation to analyse patterns. In conclusion, results showed that technology allows teachers to design experiences where graphs can be presented in a dynamical and genetic way supporting students in the transition to a meaningful managing of algebraic language. In fact, there is need for further research to explore how math teachers independently can integrate motion sensors into their classrooms and to compare the effectiveness of the use of motion sensors in relation to traditional math classroom teaching.

The keywords:
Motion sensors, math classroom, meaning of slope, rate of change, algebraic language

P51: Invariant Region Method (IRM) Applied to Two Dimensional Predator-Prey Systems

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The extended abstract:

There is a multitude of predator-prey models in literature, beginning with the models of Lotka (1925) and Volterra (1931) and have since undergone many refinements. Many models that exhibit limit cycle behaviour have been proposed in literature, but the proof of the existence is generally considered hard as one has to resort to Dulac-, Lyapunov functions and performing Liénard transformations.

The simplest mathematical proof of the existence of a limit cycle comes from the well know Poincaré-Bendixson theorem, which requires an invariant region for trajectories and an unstable critical point within that region.

A model that subsumes many of those covered in literature given by

$dx/dt = \varphi(x)g(x)-p(x)\xi(y)$
$dy/dt = \eta(y)(-\gamma +q(x)+\psi(y))$

is used to introduce a technique that shows that the solution trajectories of this model remain bounded under certain conditions. Once the equilibrium point in the interior of the population quadrant is shown to be unstable, along with the boundedness of solutions, it is known that at least one limit cycle exists. The invariant region method (IRM) and eigenvalue analysis avoids using advanced mathematical techniques and are therefore suitable for use by undergraduate students.

The keywords:
Predator-prey models, invariant region method, limit cycle, eigenvalue analysis

P52: Development of pedagogical tools for teaching mathematics - from the PC Lab and TI-92 to Smartphone, Tablet and Netbooks

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Austria

The extended abstract:

At the end of the 1980s some new universal computer algebra systems (CAS) like Mathematica (Wolfram Research) and the follower of MuMath – Derive – were introduced. Using such tools was suddenly no eccentric hobby for mathematicians at university anymore but became conceivable and available for school teachers. At the same time the Ministry for Education reequipped secondary schools in Austria with DOS-compatible and sufficiently powerful Computers. In an educational environment with high freedom of teaching like the Austrian school system a group of interested school teachers to try the new possibilities in their own math-classes was quickly found. 20 years ago in the year 1992 the first ACDCA-conference took place in Krems an der Donau in Lower Austria. The Austrian teachers presented a summary of their experiences and first results of research. The Austrian community got in touch with international experts from school and university areas and at the same time consulted with software-developers. The combination of school-practitioners, didactical researchers, international experts and the feedback of software development were the basis of all further CAS-Projects in Austria. Many aspects of the approaches have not changed much since the beginning. The impact of CAS in concept formation, development of archetypes and problem-solving was quite undisputed and accepted. What has changed since then? The computers have seen an unbelievable increase in performance and at the same time minimization. The term One-to-One-Computing took a new meaning first with handheld computers like the TI-92 from Texas Instruments up to Smartphones, Tablet-PCs and Net- or Notebooks of today. These technical changes opened the field of activity and changed the way not only how to use technology but also how to teach in the classroom. The position of the student, experimental and cooperative learning became more and more the centre of attention when using technology. Concentration on self-contained learning and new innovative learning environments (for instance following Heinz Klippert) was the implication in the following CAS-Projects. Beside CAS other technologies like dynamic geometry, spread sheets and some Internet-Tools were established as essential tools and media in mathematic teaching. The projects of the last years took place under the name "Medienvielfalt" (variation of media and tools) together with other Austrian and German research groups like GeoGebra, mathe online and mathematic-digital. Educational pathways (online content) with a focus on different tools and approaches tried to get teachers acquainted with the use of technology. Other objectives were the different approaches of boys and girls in using technology and the use of technology in exam situations. Three questions are the focus of the discussion today: 1) How to establish the implementation of qualified technology use in mathematical teaching in a compulsory and widespread way in Austria. 2) How to connect the use of technology with the project of a central competence oriented final exam in mathematics in Secondary schools. 3) How to adapt to the rapid changes of technology – especially to the use of Smartphones and Tablet-Computers.

The last 20 years of Austrian CAS-Projects brought a lot of answers, but created almost more questions – for further work there is no shortage.

P53 : Computer Algebra as an Educational Tool

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The extended abstract:

The turbulent development of computer software in mathematics requires an extension of the education of engineers. Hence, not only praxis-oriented knowledge must be supported but also the mechanics to successfully use these systems. The computer algebra systems (CAS) have improved the mathematical work of engineers. The systems are used for numerical computations as well as for algebraic manipulations of equations. Moreover, the powerful graphical capabilities and the easy use of the graphics are applied to display complicated functions and technical results. The techniques in hand calculations are trusted into the background in favor of the systematic approach in mathematics and of the exciting modeling of realistic systems. This new, exciting aspect has been taken up and the CAS Maple was included in the education of engineers. Mathematical concepts are motivated in a clear and vivid manner by the use of the visualization and animation capabilities of Maple.

In this paper the principal concept and the application of Maple in engineering education will be demonstrated in various examples:

- Lengthy and abstract topics like the convergence of Fourier series to a given function are discussed.
- The visualization of the wave equation in case of a vibrating string is performed.
- Finally, the oscillations of an idealized skyscraper are computed to visualize the meaning of eigen-values and eigenvectors.

For each of these examples a worksheet can be used interactively.

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The keywords:
Education, CAS, Maple, mathematics with Maple, mathematics for engineers

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